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# TRANSLATION

CALCULATION OF PROCESSES IN THE COMBUSTION CHAMBER AND  
NOZZLE OF A LIQUID-PROPELLANT ROCKET ENGINE

By A. V. Bolgarskiy

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A. V. Bolgarskiy

Raschet Protsessov V Kamere Sgoraniya  
i sople Zhidkostnogo Raketnogo Dvigatelya

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This book is devoted to questions of combustion and discharge at high temperatures in a liquid-propellant rocket engine. It gives the detailed technique for the thermodynamic calculation of the working process, illustrated by the solution of a series of practical problems. This book offers the bases for the possibility of applying a graphical method for calculating the parameters of the liquid-propellant rocket engine, which greatly facilitates this calculation. It gives a detailed exposition of the technique of constructing such nomograms. Two nomograms for two types of propellant, constructed by the author, are appended at the end of the book.

This book is intended for students of higher institutes of aviation who are studying the theory of combustion and exhaust, but it may also be useful to engineers working in this field.

Critics: Instructor Yu.K.Zastela,

Professor A.V.Kvasnikov, Doctor of Technical Sciences

Editor: Engineer G.Yu.Yanovskiy

Supervisor of Editorial Staff, Engineer A.I.Sokolov

## PREFACE

In this book we give an exposition of the method developed by the author for calculating the processes of fuel combustion and the discharge of the combustion products from liquid-propellant rocket engines. The entire calculation is based on the use of the conventional equation of energy, which is known to the student from his course in aerodynamics. The description of the proposed method is accompanied by a detailed exposition of the computational procedure, and by examples.

The diagrams at the end of the book (Appendixes V, VI, and VII) considerably accelerate the calculations and may be useful, especially in the preliminary rough calculations. If these diagrams do prove useful and find practical application, it will be possible to organize the calculation and construction of similar diagrams for all available fuels of liquid-propellant rocket engines.

This book will be useful not only to students in studying the theory and design of liquid-propellant rocket engines, but also to engineers working in this field.

The author expresses his thanks to Instructor V.Ye.Alemasov, Candidate in Technical Sciences, for his help in the preparation of the manuscript, and for his permission to use the Tables (Appendixes I, II, III and IV) which have been carefully recalculated and checked by him.

The author also notes the work by L.V.Ignat'yeva, laboratory assistant of the Department of Heat Engines, for arrangement of the computational diagrams and in illustrating this book, and expresses his thanks to her.

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The author will be very grateful for any comments and suggestions, addressed to  
OBORONGIZ Moscow, I-51, Petrovka, 24.

## INTRODUCTION

The appearance of a new type of thermal engine, the liquid-propellant rocket engine, demanded the development of new methods of thermal calculation since the processes of conversion of chemical energy into thermal energy and then into kinetic energy, in this engine, have certain peculiarities which complicate the calculations.

The theoretical combustion temperatures in these engines are considerably higher than in other types of thermal engines, since the oxidizers used are liquids richer in free oxygen than atmospheric air. Indeed, with the theoretically necessary quantity of oxidizer, the weight of the combustion products of 1 kg of kerosene is, in round numbers, as follows: with atmospheric air as oxidizer, 16 kg; with hydrogen peroxide, 8.5 kg; with nitric acid, 6.3 kg; and with liquid oxygen, 4.5 kg. And yet the quantity of heat liberated on combustion is almost the same with these different oxidizers. It is entirely understandable that, with a smaller quantity of combustion products, they must be heated to a higher temperature.

The high theoretical combustion temperature causes a more extensive dissociation in the combustion chamber and a partial recombination of the molecules as the gases flow into the exhaust nozzle where, on expansion, the temperature of the gases drops. The need to take these phenomena into account makes it more difficult to handle the thermal calculation.

On the other hand, there are no fundamentally new phenomena in the thermal processes in liquid-propellant rocket engines, and consequently, the entire thermal calculation should be founded on the propositions generally adopted in thermal

technology.

The book by A.V.Bolgarskiy and V.K.Shchukin ("Working Processes in Liquid-Propellant Rocket Engines", Oborongiz, 1953) gives the principles of a method of calculating the thermal processes for liquid-propellant rocket engines, based on the general propositions and concepts of thermal technology and of thermal engines.

The present work represents an expanded and detailed exposition of the calculation method which is a logical development of general thermal calculations extended to a new type of thermal engine.

On the basis of the fact that all the processes taking place in the combustion chamber and nozzle of the liquid-propellant rocket engine are conversions of energy from one form to another (chemical - thermal - kinetic), the author bases his entire technique on the usual equation of energy (cf. Chapter II); in this equation, from the total quantity of heat  $Q$  he separates the heat obtained on combustion of the fuel.

Only the simplest propellants, in widest use, are considered in this book. They are chemical compounds of the systems C, H, O, and N.

The application of the proposed calculation method to fuels of other types, and the construction of computational diagrams for other fuels to be used, is a task for the future.

The calculation per mole of fuel has been taken as the basis of exposition of the method.



## CHAPTER I

### PROPELLANT CALCULATIONS

#### 1. Chemical Composition of the Components

The fuel for liquid-propellant rocket engines consists of a combustible and an oxidizer. The combustibles ordinarily employed are either hydrocarbons or alcohols; kerosene, ethyl and methyl alcohols are most often used. The usual oxidizers are nitric acid, liquid oxygen, or hydrogen peroxide.

Hereafter we shall take  $C_n H_m O_p$  as the general chemical formula for the combustible and  $H_t N_u O_v C_q$  for the oxidizer.

If the components are specified by chemical formulas, then their molecular weight is calculated by the formulas

$$\begin{aligned}\mu_c &= 12n + m + 16p; \\ \mu_o &= t + 14u + 16v + 12q.\end{aligned}\tag{1}$$

The conversion to elementary composition by weight is performed according to the formulas

$$\begin{aligned}C_c &= \frac{12n}{\mu_c} 100\%, \\ H_c &= \frac{m}{\mu_c} 100\%, \\ O_c &= \frac{16p}{\mu_c} 100\%\end{aligned}\tag{2}$$

and by analogy, for the oxidizer,

$$H_o = \frac{t}{\mu_o} 100\%,$$

$$N_o = \frac{14u}{\mu_o} 100\%, \quad (3)$$

$$O_o = \frac{16v}{\mu_o} 100\%,$$

$$C_o = \frac{12q}{\mu_o} 100\%.$$

If the components are specified by elementary composition by weight, i.e.:

Combustible -  $C_c\%$ ,  $H_c\%$ ,  $O_c\%$ ,

Oxidizer -  $H_o\%$ ,  $O_o\%$ ,  $C_o\%$ ,  $N_o\%$ ,

then the calculation for finding the chemical formula is performed as follows: The number of atoms of hydrogen are taken as equal to its percentage content in the component, and then the number of atoms of the remaining elements in the component are calculated. Thus, the combustible contains  $\frac{C_c}{12}$  atoms of carbon,  $\frac{O_c}{16}$  atoms of oxygen, while the oxidizer contains  $\frac{N_o}{14}$  atoms of nitrogen,  $\frac{C_o}{12}$  atoms of carbon and  $\frac{O_o}{16}$  atoms of oxygen; we then get the following arbitrary formulas:

$$\text{Combustible} - C_{\frac{C_c}{12}} H_{\frac{H_c}{1}} O_{\frac{O_c}{16}}. \quad (4)$$

$$\text{Oxidizer} - H_{\frac{H_o}{1}} N_{\frac{N_o}{14}} O_{\frac{O_o}{16}} C_{\frac{C_o}{12}}.$$

In this case the arbitrary molecular weight is taken as equal to 100.

Very often the components of a propellant do not constitute 100% of the substance, but have been diluted with water (for instance ethyl and methyl alcohols, nitric acid, hydrogen peroxide). We allow for the presence of water in the components by their concentrations, expressing the percentage content of the pure substance in a mixture with water; for example, if the concentration of hydrogen peroxide is 80%, this means that 1 kg of this component contains 0.8 kg of pure hydrogen peroxide and 0.2 kg of water.

If the component is to be expressed by a chemical formula, then it is more convenient to express the quantity of water by the number of moles of water to 1 mole of the substance.

Let the concentration of the substance be  $\sigma\%$ , and its molecular weight be  $\mu$ ; then the calculation is conducted as follows: 1 kg of component contains  $\frac{\sigma}{100}$  kg of the pure substance and  $\frac{100 - \sigma}{100}$  kg of water; and it contains the following quantity of water per 1 kg of pure substance:

$$\frac{100 - \sigma}{100} : \frac{\sigma}{100} = \frac{100 - \sigma}{\sigma} \text{ kg/kg.}$$

There is  $\frac{100 - \sigma}{\sigma} \mu$  kg of water per mole of pure substance; expressing this quantity of water in moles and remembering that the molecular weight of water is  $\mu_w = 18$ , we get:

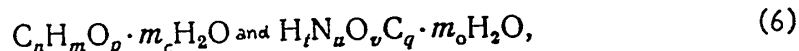
$$m = \frac{100 - \sigma}{\sigma \mu_w} \mu = \frac{100 - \sigma}{18\sigma} \mu \text{ mole/mole} \quad (5)$$

Table 1 gives the values of  $m$  for components most frequently encountered in concentrations below 100%.

Table 1

Type of Component	Molecular Weight	Concentration in %					
		95	90	85	80	75	70
Ethyl alcohol	46	0,135	0,284	0,451	0,639	0,852	1,095
Methyl alcohol	32	0,094	0,198	0,314	0,445	0,593	0,762
Nitric acid	63	0,184	0,389	0,618	0,875	1,167	1,500
Hydrogen peroxide	34	0,099	0,209	0,333	0,472	0,630	0,809

In the symbols used above, the chemical formulas of components with the concentration  $\sigma_c\%$  and  $\sigma_o\%$  will be of the form



where

$$m_c = \frac{100 - \sigma_c}{18\sigma_c} \mu_c, \quad m_o = \frac{100 - \sigma_o}{18\sigma_o} \mu_o.$$

The weight of 1 mole of the component, of a concentration  $\sigma\%$ , will hereafter be termed the molecular weight of the component and will be designated by  $\mu'$ . At any concentration, this weight will be determined as follows:

$$\mu' = \mu + 18m = \mu + 18\mu \frac{100 - \sigma}{18\sigma} = \frac{100\mu}{\sigma} \text{ Kg/mole} \quad (7)$$

Table 2 gives the values of  $\mu'$  for the most frequently used components.

Table 2

Component	Chemical Formula	Molecular Weight at Concentration $\sigma\%$ :						
		100	95	90	85	80	75	70
Ethyl alcohol	$C_2H_6O$	46	48,4	51,1	54,1	57,5	61,3	65,7
Methyl alcohol	$CH_4O$	32	33,6	35,5	37,7	40,0	42,6	45,7
Nitric acid	$HNO_3$	63	66,3	70,0	74,1	78,8	84,0	90,0
Hydrogen peroxide	$H_2O_2$	34	35,7	37,7	40,0	42,5	45,3	48,6

For a component expressed by a chemical formula, at a concentration below 100%, its elementary composition by weight is calculated as follows.

Let a combustible be specified by the chemical formula  $C_n N_m O_p \cdot m_c H_2O$ .

The number of atoms of the various elements per mole of combustible and their weights will be determined as follows:

Carbon	-	number of atoms	-	n	weight	12 n
Hydrogen	-	"	"	"	-	m + 2m <sub>c</sub>
Oxygen	-	"	"	"	-	p + m <sub>c</sub>
						16(p + m <sub>c</sub> )

Total weight of 1 mole of combustible  $12n + m + 16p + 18m_c$ .

On the basis of eqs.(1) and (7) we find

$$12n + m + 16p + 18m_c = \mu_c + 18m_c = \mu'_c.$$

Consequently, the elementary composition of the combustible by weight [cf. eq.(2)] will be expressed by the following relations:

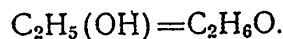
$$\begin{aligned}
 C'_c &= \frac{12n}{\mu_c} 100\%, \\
 H'_c &= \frac{m + 2m_c}{\mu_c} 100\%, \\
 O'_c &= \frac{16(p + m)}{\mu_c} 100\%.
 \end{aligned} \tag{8}$$

Similarly, we get for the oxidizer:

$$\begin{aligned}
 C'_o &= \frac{12q}{\mu_o} 100\%, \\
 H'_o &= \frac{t + 2m_o}{\mu_o} 100\%, \\
 O'_o &= \frac{16(v + m_o)}{\mu_o} 100\%, \\
 N'_o &= \frac{14u}{\mu_o} 100\%.
 \end{aligned} \tag{8*}$$

Example 1. Calculate the elementary composition, by weight, of ethyl alcohol, at concentrations of 60, 80, and 100%.

The chemical formula of pure ethyl alcohol is as follows:



Consequently, its molecular weight will be

$$\mu_c = 12 \cdot 2 + 1 \cdot 6 + 16 \cdot 1 = 46.$$

The number of molecules of water per mole of the pure substance at the specified concentrations is calculated on the basis of eq.(5) and will be

$$m_c = \frac{100 - \sigma_c}{18\sigma_c} \mu_c = \frac{100 - \sigma_c}{\sigma_c} \cdot \frac{46}{18} = 2,555 \frac{100 - \sigma_c}{\sigma_c}.$$

At the specified concentrations, this formula yields

$$\begin{aligned}
 m_c^{100} &= 0, \\
 m_c^{80} &= 0,639, \\
 m_c^{60} &= 1,703.
 \end{aligned}$$

The weight of 1 mole of the component, of a concentration  $\sigma\%$ , will hereafter be termed the molecular weight of the component and will be designated by  $\mu'$ . At any concentration, this weight will be determined as follows:

$$\mu' = \mu + 18m = \mu + 18\mu \frac{100 - \sigma}{18\sigma} = \frac{100\mu}{\sigma} \text{ Kg/mole} \quad (7)$$

Table 2 gives the values of  $\mu'$  for the most frequently used components.

Table 2

Component	Chemical Formula	Molecular Weight at Concentration $\sigma\%$ :						
		100	95	90	85	80	75	70
Ethyl alcohol	$\text{C}_2\text{H}_5\text{O}$	46	48,4	51,1	54,1	57,5	61,3	65,7
Methyl alcohol	$\text{CH}_3\text{O}$	32	33,6	35,5	37,7	40,0	42,6	45,7
Nitric acid	$\text{HNO}_3$	63	66,3	70,0	74,1	78,8	84,0	90,0
Hydrogen peroxide	$\text{H}_2\text{O}_2$	34	35,7	37,7	40,0	42,5	45,3	48,6

For a component expressed by a chemical formula, at a concentration below 100%, its elementary composition by weight is calculated as follows.

Let a combustible be specified by the chemical formula  $\text{C}_n\text{N}_m\text{O}_p \cdot m_c\text{H}_2\text{O}$ .

The number of atoms of the various elements per mole of combustible and their weights will be determined as follows:

Carbon	-	number of atoms	-	n	weight	12 n
Hydrogen	-	"	"	"	-	m + 2m <sub>c</sub>
Oxygen	-	"	"	"	-	p + m <sub>c</sub>
						16(p + m <sub>c</sub> )

Total weight of 1 mole of combustible  $12n + m + 16p + 18m_c$ .

On the basis of eqs.(1) and (7) we find

$$12n + m + 16p + 18m_c = \mu_c + 18m_c = \mu'_c.$$

Consequently, the elementary composition of the combustible by weight [cf. eq.(2)] will be expressed by the following relations:

The corresponding chemical formulas will have the form

$$\begin{aligned} \text{at } \sigma_c &= 100\% - \text{C}_2\text{H}_6\text{O}, \\ \text{at } \sigma_c &= 80\% - \text{C}_2\text{H}_6\text{O} \cdot 0,639\text{H}_2\text{O}, \\ \text{at } \sigma_c &= 60\% - \text{C}_2\text{H}_6\text{O} \cdot 1,703\text{H}_2\text{O}. \end{aligned}$$

On the basis of eq.(7), the molecular weight of alcohol is

$$\begin{aligned} \text{at } \sigma_c &= 100\% & \mu'_c &= \mu_c = 46, \\ \text{at } \sigma_c &= 80\% & \mu'_c &= \frac{100 \cdot 46}{80} = 57,5, \\ \text{at } \sigma_c &= 60\% & \mu'_c &= \frac{100 \cdot 46}{60} = 76,67. \end{aligned}$$

We recall that the term molecular weight of a substance of lower concentration ( $\sigma < 100$ ) means the sum of the molecular weight of the pure substance and the weight of the water in 1 mole of the substance; for example, at a 60% concentration of alcohol, the weight of 1 mole will consist of the weight of 1 mole of pure alcohol  $\mu_c = 46$  and of the weight of  $m_c$  moles of water; since  $m_c = 1.703$ , the molecular weight of this alcohol will be

$$\mu' = \mu + 18m_c = 46 + 18 \cdot 1,703 = 76,67 \text{ kg}$$

The composition of alcohol by weight at the concentrations is found from eq.(8) to be as follows:

	$C_c$	$H_c$	$O_c$
$\sigma_c = 100\%$	$\frac{12 \cdot 2}{46} 100 = 52,1\%$	$\frac{6}{46} 100 = 13,1\%$	$\frac{16 \cdot 1}{46} 100 = 34,8\%$
$\sigma_c = 80\%$	$\frac{12 \cdot 2}{57,5} 100 = 41,8\%$	$\frac{6 + 2 \cdot 0,639}{57,5} 100 = 12,6\%$	$\frac{16(1 + 0,639)}{57,5} 100 = 45,6\%$
$\sigma_c = 60\%$	$\frac{12 \cdot 2}{76,67} 100 = 31,3\%$	$\frac{6 + 2 \cdot 1,703}{76,67} 100 = 12,3\%$	$\frac{16(1 + 1,703)}{76,67} 100 = 56,4\%$

## 2. The Heat Value of the Components

In calculating the combustion process it is necessary to know the quantity of

chemical energy introduced into the combustion chamber by the components of the propellant: in technical calculation, this chemical energy is termed the heat value. In engines that use atmospheric air as the oxidizer, it is customary to term the combustible "fuel" and in calculating the combustion processes the heat value of the combustible is called the heat value of the fuel, since the heat value of atmospheric air is zero.

In calculating the combustion processes in liquid-propellant rocket engines it is necessary to take account of both the heat value of the combustible and the heat value of the oxidizer. The quantity of heat that can be liberated on complete consumption of 1 kg of a component during the combustion process is taken as the chemical energy, i.e., the heat value, of that component. As a general rule, the heat value of a component may be determined by the Hess law, starting out from the proposition that the component at first must be decomposed into its constituent elements, for which a quantity of heat, equal to the heat of formation, but taken with reversed sign, must be expended, and that the combustible elements of the component must then burn with total liberation of the chemical energy (heat value) in the form of heat.

Further than this, one must also take account of the presence of water in the components. A certain quantity of heat must be expended to evaporate this water, since the lowest heat value of the component must enter into the technical calculations. The heat value of the combustible component elements may be taken from Table 4 (see below), while the consumption of heat in evaporating the moisture in the components is equal to 10,500 kcal per mole of water.

All calculations on combustion processes are made at the initial temperature of 25°C (298° abs), since it is customary to give the values of the thermal effect of reactions at this temperature. At this temperature, the consumption of heat to evaporate 1 mole of water is expressed in round numbers at 10,500 kcal/mole; in actuality, according to steam Tables (see M.P.Vukalovich and I.I.Novikov: "Technical



Thermodynamics"), the latent heat of vaporization is 583.2 kcal/kg; consequently it is necessary to expend

$$583,2 \times 18 = 10497,6 \approx 10\,500 \text{ kcal / mole}$$

on the evaporation of 1 mole of water at 25°C.

Under these conditions, the formulas for determining the heat value of the components now take the form:

Combustible

$$H_{p_c} = 94\,050n + 57\,800 \cdot 0,5m - H_{form} - 10\,500m_c, \quad (9)$$

Oxidizer

$$H_{p_o} = 94\,050q + 57\,800 \cdot 0,5t - H_{form} - 10\,500m_o.$$

The heat value of the components per kg is determined by the formulas

$$\begin{aligned} H_c &= \frac{H_{p_c}}{p_c} = \frac{H_{p_c} \varepsilon_c}{100 p_c} \text{ kcal / kg}, \\ H_o &= \frac{H_{p_o}}{p_o} = \frac{H_{p_o} \sigma_o}{100 p_o} \text{ kcal / kg}. \end{aligned} \quad (10)$$

In reality, the heat value of the combustible is usually known.

The values  $H_{form}$  for the components is as follows:

For 100% nitric acid,  $H_{form} = 41.400 \text{ kcal/mole}$ ,

For 100% hydrogen peroxide,  $H_{form} = 44.840 \text{ kcal/mole}$ .

In the case of components with concentrations below 100%, the heat of solution\* must be taken into account; for hydrogen peroxide it is slight (11 kcal/kg) and for other oxidizers it may be calculated by the formula:

$$H_{sol} = \frac{Am}{\alpha + m},$$

where m is the number of moles of water per mole of pure substance; for nitric acid,

$A = 8974$ ,  $\alpha = 1.737$ .

The sum  $H_{form} + H_{sol}$  is usually included in the calculation of the heat value;

\*The heats of solution for several substances are given in Appendix IV.

for nitric acid this sum, depending on the concentration, is:

$\sigma_o\%$	100	95	90	85	80
$H_{form} + H_{sc}$	41 400	42 260	43 040	43 760	44 410

*Kcal /mole*

The heat value of oxidizers may be either positive ( $H_2O_2$ ,  $CN_4O_8$ ) or negative ( $HNO_3$ ).

As stated above, the heat value for combustibles is usually known. Table 3 gives the heat value of ethyl and methyl alcohol as a function of the concentration.

Table 3

Concentration in %	Ethyl Alcohol		Methyl Alcohol	
	<i>Kcal /mole</i>	<i>Kcal /kg</i>	<i>Kcal /mole</i>	<i>Kcal /kg</i>
100	296 200	6440	149 700	4680
95	294 780	6080	148 710	4410
90	293 200	5740	147 620	4150
85	291 440	5380	145 010	3890
80	289 450	5030	145 010	3620
75	287 200	4680	143 440	3360
70	284 650	4330	141 660	3090
65	281 680	3980	139 600	2830
60	278 220	3630	137 200	2570

In this Table the values of  $H_c$  have been calculated for given values of  $H_{\mu c}$  by the formula:

$$H_c = \frac{H_{\mu c} \sigma_c}{100 \mu_c} \text{ Kcal /kg.}$$

For kerosene the heat value by weight ranges from 10,200 to 10,500 kcal/kg.

Equation (9) for determining the heat value of oxidizers gives the following heat values for nitric acid and hydrogen peroxide at various concentrations:

	$\sigma_0$ , %	100	90	80	70
HNO <sub>3</sub>	Kcal / mole	-12 500	-16 680	-21 770	—
	Kcal / Kg	-198	-237	-278	—
H <sub>2</sub> O <sub>2</sub>	Kcal / mole	12 600	10 380	7600	4020
	Kcal / Kg	371	275	179	83

### 3. Specific Gravity of the Components

The specific gravity of the components is determined as the ratio of their weight at a definite temperature (for hydrocarbons usually at 20°C) to the weight of the same volume of water at 4°C; this specific gravity is a dimensionless quantity and is denoted by  $d_4^{20}$ , where the subscript "4" indicates that the water is taken at a temperature of 4°C, while the superscript "20" means that the component is taken at 20°C. Since the weight of 1 liter of water at 4°C is 1 kg, the dimensionless value of the specific gravity  $d$  is equal to the density  $\gamma$  adapted in technology, which has the dimension of kg/ltr; this quantity is likewise customarily termed the specific gravity in technology.

The specific gravity of petroleum products depends significantly on the temperature; if the specific gravity of a petroleum product at 20°C is  $\gamma_0$ , then, at a temperature  $t$ °C, it may be determined by the formula:

$$\gamma_t = \gamma_0 - \alpha(t - 20).$$

In this formula the value of the coefficient  $\alpha$  is determined as a function of the specific gravity of the petroleum product, by the formula:

$$\alpha = 0,000581 + 0,00126(0,95 - \gamma_0).$$

The specific gravity of 100% alcohols is determined as a function of the temperature according to the formulas:

$$\text{For ethyl alcohol: } \gamma_t = 0,80665 - 0,000866t;$$

For methyl alcohol:  $\gamma_s = 0,81034 - 0,000917t$ .

The specific gravity of alcohols at 25°C may be determined approximately as a function of their concentration by the following formulas:

For ethyl alcohol:  $\gamma_s = 0,8623 - 0,00253(\sigma - 70)$ ;

For methyl alcohol:  $\gamma_s = 0,8658 - 0,00267(\sigma - 70)$ .

The temperature dependence and the concentration dependence of the specific gravity of 100% nitric acid at 25°C is expressed by the following formulas:

In the range from 0 to 80°  $\gamma_s = 1,53 - 0,0014t$ ;

In the range from 90 to 100%  $\gamma_s = 1,492 - 0,0012(\sigma - 90)$ .

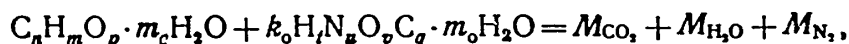
The specific gravity of liquid oxygen may be taken as  $\gamma = 1.14$  kg/ltr.

The specific gravity of hydrogen peroxide at 25°C, as a function of the concentration, is

$$\gamma_s = 1,16 + 0,005(\sigma - 40).$$

#### 4. Stoichiometric Coefficient of the Ratio of the Components and the Coefficient of Excess Oxidizer

The stoichiometric coefficient of the ratio of the components is the term applied to the number of moles of oxidizer theoretically necessary for the total combustion of 1 mole of combustible. The stoichiometric coefficient of the ratio of the components is simplest determined from the equations of the chemical reactions. If the combustible is defined by the chemical formula  $C_n H_m O_p \cdot m_c \cdot H_2O$ , and the oxidizer by the chemical formula  $H_t N_u O_v C_q \cdot m_o H_2O$ , then in the general form, the process of complete combustion of the propellant will be represented by the formula:



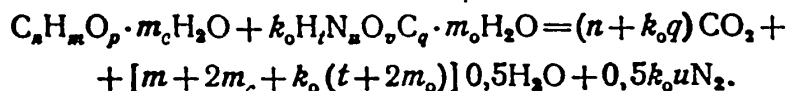
where  $M_{CO_2}$ ,  $M_{H_2O}$ , and  $M_{N_2}$  are the respective numbers of moles of these gases in the combustion product, while  $k_o$  is the stoichiometric coefficient of the ratio of the components.

The number of moles of carbon dioxide, water vapor, and nitrogen may be de-

terminated from the number of atoms of the elements in the propellant. The total number of atoms of carbon in the propellant is  $n + k_o q$ , that of hydrogen is  $m + 2m_c + k_o(t + 2m_o)$ ; and that of nitrogen is  $k_o u$ ; bearing in mind that a molecule of carbon dioxide has one atom of carbon, and molecules of water vapor and nitrogen each have two atoms of hydrogen and nitrogen respectively, we may write:

$$\begin{aligned} M_{CO_2} &= n + k_o q; \\ M_{H_2O} &= 0,5 [m + 2m_c + k_o(t + 2m_o)]; \\ M_{N_2} &= 0,5 k_o u. \end{aligned} \quad (11)$$

Thus the total chemical equation of the combustion process will have the form:



The stoichiometric coefficients of the ratio of the components in this case will be determined from the condition that the number of atoms of oxygen in the propellant must equal the number of atoms of oxygen in the combustion products; from the fundamental equation, in this case, we get

$$p + m_c + k_o v + k_o m_o = 2(n + k_o q) + 0,5 [m + 2m_c + k_o(t + 2m_o)].$$

The solution of this equation for the quantity  $k_o$  yields

$$k_o = \frac{2n + 0,5m - p}{v - 2q - 0,5t} \text{ mole / mole} \quad (12)$$

As was to be expected, the concentration of the components has no effect on the value of the stoichiometric coefficient. The weight stoichiometric coefficient of the component ratio is determined by the molecular weights of the components, more specifically:

$$k'_o = k_o \frac{\mu_o}{\mu_c} = k_o \frac{\mu_o}{\mu_c} \frac{\sigma_c}{\sigma_o} K_g / K_g. \quad (13)$$

The bulk modulus of the ratio of the components will be equal to

$$k'_o = k'_o \frac{\gamma_c}{\gamma_o} = k_o \frac{\mu_o}{\sigma_o} \frac{\sigma_c}{\sigma_o} \frac{\gamma_c}{\gamma_o} \text{ hr/ltr} \quad (14)$$

If the components are specified by their elementary composition by weight, namely: the combustible by  $C_c\%$ ,  $H_c\%$ ,  $O_c\%$ , and the oxidizer by  $H_o\%$ ,  $N_o\%$ ,  $O_o\%$ ,  $C_o\%$ , then, for water-free components, we find that the total combustion of 1 kg of combustible requires  $\frac{\frac{8}{3} C_c + 8H_c - O_c}{100}$  kg of oxygen in this case, while 1 kg of oxidizer contains  $\frac{O_o - \frac{8}{3} C_o - 8H_o}{100}$  kg of free oxygen. Consequently, the weight stoichiometric coefficient of the ratio of the components will be

$$k'_o = \frac{\frac{8}{3} C_c + 8H_c - O_c}{O_o - \frac{8}{3} C_o - 8H_o} \text{ kg/kg.} \quad (15)$$

At concentrations  $\sigma_c\%$  and  $\sigma_o\%$  of the components, respectively, the combustion of 1 kg of combustible will require only

$$\frac{\frac{8}{3} C_c + 8H_c - O_c}{100} \frac{\sigma_c}{100} \text{ kg,}$$

of oxygen, and 1 kg of oxidizer will contain only

$$\frac{O_o - \frac{8}{3} C_o - 8H_o}{100} \frac{\sigma_o}{100} \text{ kg,}$$

of oxygen and, consequently,

$$k'_o = \frac{\frac{8}{3} C_c + 8H_c - O_c}{O_o - \frac{8}{3} C_o - 8H_o} \frac{\sigma_c}{\sigma_o} \text{ kg/kg,} \quad (16)$$

and since, according to the preceding,

$$k'_o = k_o \frac{\mu_o}{\mu_c} \frac{\sigma_c}{\sigma_o},$$

then

$$k_o = k'_o \frac{\mu_c}{\mu_o} \frac{\sigma_o}{\sigma_c} = \frac{\frac{8}{3} C_c + 8 H_c - O_c}{O_o - \frac{8}{3} C_o - 8 H_o} \frac{\mu_c}{\mu_o} \quad (17)$$

Example 2. Propellant: toluene ( $C_7H_8$ ) and 95% nitric acid ( $HNO_3 \cdot 0.184 H_2O$ ). To find the molar and weight stoichiometric coefficients of the ratio of the components:

The molar stoichiometric coefficient is determined by eq.(12):

$$k_o = \frac{2 \cdot 7 + 0,5 \cdot 8}{3 - 0,5} = 7,2 \text{ mole / mole}$$

Consequently, at  $\sigma_o = 95\%$ , the weight stoichiometric coefficient, on the basis of eq.(13), is found to be

$$k'_o = 7,2 \frac{63}{92} \cdot \frac{100}{95} = 5,19 \text{ kg/kg.}$$

Example 3. Weight composition of ethyl alcohol:  $C_c = 52.2\%$ ,  $H_c = 13\%$ ,  $O_c = 34.8\%$ , and of hydrogen peroxide:  $H_o = 5.9\%$ ,  $O_o = 94.1\%$ . The concentration of the alcohol is  $\sigma_c = 90\%$  and that of the hydrogen peroxide,  $\sigma_o = 80\%$ . The specific gravity of ethyl alcohol of the specified concentration at  $25^\circ C$  is  $\gamma_c = 0.812 \text{ kg/ltr}$ , and that of the hydrogen peroxide is  $\gamma_o = 1.35 \text{ kg/ltr}$ . To find the molar, weight, and volume stoichiometric coefficients:

The weight stoichiometric coefficient is found from eq.(16):

$$k'_o = \frac{2,667 \cdot 52,2 - 8 \cdot 13 - 34,8}{94,1 - 8 \cdot 5,9} \cdot \frac{90}{80} = 4,955 \text{ kg/kg}$$

Consequently,

$$k_o = 4,955 \cdot \frac{\mu}{\mu_o} \frac{\sigma_o}{\sigma} = 4,955 \cdot \frac{46}{34} \cdot \frac{80}{90} = 6 \text{ mole / mole}$$

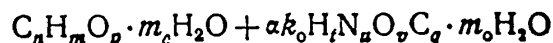
The volume stoichiometric coefficient will be

$$k''_o = k'_o \frac{\gamma}{\gamma_o} = 4,955 \cdot \frac{0,822}{1,35} = 3,04 \text{ ltr/ltr}$$

The above stoichiometric coefficients of the ratio of the components determine the theoretically necessary quantity of oxidizer for the complete combustion of 1 mole, 1 kg, or 1 ltr of combustible. In reality, the quantity of oxidizer supplied to the combustion chamber is not equal to the calculated theoretical quantity, and most often it is less than this quantity; such a ratio of the components lowers the efficiency, but does increase the specific thrust of the engine. The ratio between the quantities of components in this case is evaluated by the coefficient of excess oxidizer  $\alpha$ , which determines the actual quantity of oxidizer per mole of combustible, in fractions of the theoretically necessary quantity.

##### 5. Weight and Volume Composition of a Propellant

Taking account of the coefficient of excess oxidizer adopted, the composition of the propellant before combustion will be determined by the formula:



and per mole of combustible it will have the following weight:

$$\mu'_c + \alpha k_o \mu'_o \text{ Kg.}$$

The weight composition of the propellant will be determined by the following formulas:

$$g_c = \frac{\mu'_c}{\mu'_c + \alpha k_o \mu'_o}, \quad g_o = \frac{\alpha k_o \mu'_o}{\mu'_c + \alpha k_o \mu'_o},$$

but

$$\mu'_c = \frac{100\mu_c}{\sigma_c} \text{ and } \mu'_o = \frac{100\mu_o}{\sigma_o};$$

Consequently,

$$g_c = \frac{\frac{\mu_c}{\sigma_c}}{\frac{\mu_c}{\sigma_c} + \frac{\alpha k_o \mu_o}{\sigma_o}} = \frac{\mu_c \sigma_o}{\mu_c \sigma_o + \alpha k_o \mu_o \sigma_c};$$

$$g_o = \frac{\frac{\alpha k_o \mu_o}{\sigma_o}}{\frac{\mu_c}{\sigma_c} + \frac{\alpha k_o \mu_o}{\sigma_o}} = \frac{\alpha k_o \mu_o \sigma_c}{\mu_c \sigma_o + \alpha k_o \mu_o \sigma_c}.$$

(18)



If the propellant components are specified by their elementary composition by weight, then, as stated above, we determine the weight stoichiometric coefficient of the ratio of the components, and in that case,  $ak'_0$  kg of oxidizer will be consumed per kg of combustible; consequently, the composition by weight of the propellant will be

$$g_c = \frac{1}{1 + ak'_0}, \quad g_o = \frac{ak'_0}{1 + ak'_0}. \quad (19)$$

The composition by volume of the propellant is determined from the known specific gravities of the components  $\gamma_c$ , kg/ltr, and  $\gamma_o$ , kg/ltr; in this case we get

$$v = v_o + v_c = \frac{g_c}{\gamma_c} + \frac{g_o}{\gamma_o} \text{ ltr/kg}. \quad (20)$$

The specific gravity of the propellant will be

$$\gamma = \frac{1}{v} = \frac{1}{\frac{g_c}{\gamma_c} + \frac{g_o}{\gamma_o}} = \frac{\gamma_c \gamma_o}{g_c \gamma_o + g_o \gamma_c}. \quad (21)$$

The composition by volume of the propellant may be determined from these formulas as follows:

$$\begin{aligned} r_p &= \frac{v_c}{v} = \frac{g_c \gamma_o}{g_c \gamma_o + g_o \gamma_c}; \\ r_o &= \frac{v_o}{v} = \frac{g_o \gamma_c}{g_c \gamma_o + g_o \gamma_c}. \end{aligned} \quad (22)$$

Thus 1 ltr of propellant contains  $r_c$  liters of fuel and  $r_o$  liters of oxidizer. Substituting the values of  $g_c$  and  $g_o$  in these formulas, we get

$$\begin{aligned} r_c &= \frac{\mu_c \gamma_o}{\mu_c \gamma_o + ak_o \mu_o \gamma_c}; \\ r_o &= \frac{ak_o \mu_o \gamma_c}{\mu_c \gamma_o + ak_o \mu_o \gamma_c}. \end{aligned}$$

If the components are specified by their elementary composition by weight, then the volume of 1 kg of propellant will be

$$v = \frac{1}{\gamma (1 + ak'_0)} + \frac{ak'_0}{\gamma_o (1 + ak'_0)} = \frac{1}{1 + ak'_0} \left( \frac{1}{\gamma_c} + \frac{ak'_0}{\gamma_o} \right) \text{ ltr/kg}, \quad (23)$$

and its specific gravity will be

$$\gamma = \frac{(1 + \alpha k_o') \gamma_c \gamma_o}{\gamma_o + \alpha k_o' \gamma_c} \text{ kg/ltr} \quad (24)$$

The composition by volume will in this case be determined by the formulas

$$\begin{aligned} r_c &= v_c \gamma = \frac{\gamma_o}{\gamma_o + \alpha k_o' \gamma_c}; \\ r_o &= v_o \gamma = \frac{\alpha k_o' \gamma_c}{\gamma_o + \alpha k_o' \gamma_c}. \end{aligned} \quad (25)$$

If the volume stoichiometric coefficient of the ratio of the components is known, then

$$r_c = \frac{1}{1 + \alpha k_o'}, \quad r_o = \frac{\alpha k_o'}{1 + \alpha k_o'}.$$

Example 4. Determine the weight and volume characteristics of the following propellant: ethyl alcohol and 80% hydrogen peroxide. The concentration of the alcohol is 60, 80, and 100%; the specific gravities of the components are: for hydrogen peroxide,  $\gamma_o = 1.35 \text{ kg/ltr}$ , for ethyl alcohol  $\gamma_c^{100} = 0.7864 \text{ kg/ltr}$ ,  $\gamma_c^{80} = 0.837 \text{ kg/ltr}$ ,  $\gamma_c^{60} = 0.8876 \text{ kg/ltr}$ . The coefficient of excess oxidizer is  $\alpha = 0.8$ .

The stoichiometric coefficients of the ratio of the components in this case is found to be

$$k_o = \frac{2 \cdot 2 + 0.5 \cdot 6 - 1}{2 - 1} = 6 \text{ mole / mole}$$

For  $\alpha = 0.8$ , the molar ratio of the components will be equal to

$$\alpha k_o = 0.8 \cdot 6 = 4.8 \text{ mole / mole}$$

Since  $\mu_c = 46$ , and  $\mu_o = 34$ , then, by using eq.(18), we get

$$\begin{aligned} g_c &= \frac{46 \cdot 80}{46 \cdot 80 + 4.8 \cdot 34 \sigma_c} = \frac{3680}{3680 + 163.2 \sigma_c} = \frac{1}{1 + 0.04435 \sigma_c}; \\ g_o &= \frac{4.8 \cdot 34 \sigma_c}{46 \cdot 80 + 4.8 \cdot 34 \sigma_c} = \frac{163.2 \sigma_c}{3680 + 163.2 \sigma_c} = \frac{0.04435 \sigma_c}{1 + 0.04435 \sigma_c}. \end{aligned}$$

After substituting the values of  $\sigma_c$ , we get the following composition by weight:

$\sigma_c \%$	100	80	60
$g_c$	0,184	0,220	0,273
$g_o$	0,816	0,780	0,727

The specific volume and the specific weight of the propellant are determined by eqs.(20) and (21); the results of the calculations are as follows:

$\sigma_c \%$	100	80	60
$v \text{ ltr/kg}$	0,838	0,840	0,848
$\gamma \text{ kg/ltr}$	1,195	1,190	1,180

The composition of the propellant by volume is calculated by eq.(22):

$$r_c = \frac{g_c \gamma_o}{g_c \gamma_o + g_o \gamma_c} = \frac{1,35 g_c}{1,35 g_c + g_o \gamma_c};$$

$$r_o = \frac{g_o \gamma_c}{g_c \gamma_o + g_o \gamma_c} = \frac{g_o \gamma_c}{1,35 g_c + g_o \gamma_c}.$$

After substituting the corresponding values of  $g_c$ ,  $g_o$  and  $\gamma_c$ , the composition of the propellant by volume is determined for all three cases, as follows:

$\sigma_c \%$	100	80	60
$r_c$	0,279	0,313	0,363
$r_o$	0,721	0,687	0,637

The values of the coefficients of the ratio of the components may be determined, either from the composition of the propellant by weight and volume so obtained or from the formulas given earlier.

Calculation gives the following results:

$\sigma_c \%$	100	80	60
$ak_o \text{ mole/mole}$	4,8	4,8	4,8
$ak_o' \text{ kg/kg}$	4,44	3,54	2,66
$ak_o'' \text{ ltr/ltr}$	2,66	2,16	1,72

## CHAPTER II

THE EQUATION OF ENERGY IN THE PROCESSES OF COMBUSTION  
AND OF DISCHARGE OF THE COMBUSTION PRODUCTS1. Processes in the Combustion Chamber and in the Exhaust Nozzle

The processes taking place in the combustion chamber and in the exhaust nozzle consist of the combustion of the propellant at very high temperatures, and the efflux of the combustion products at very high velocities. The processes of combustion of the propellant and of the efflux of the combustion products are rather complex conversions of energy from one form to another.

The components of the propellant introduced into the combustion chamber, having a very small amount of kinetic energy (as the result of the low intake velocities) and having a slight quantity of heat in the form of enthalpy (heat content), contain a very substantial quantity of chemical energy. In the combustion chamber the combustion of this propellant takes place, i.e., the conversion of its chemical energy into thermal energy, which increases the temperature of the combustion product to very high values.

The combustion of the propellant cannot be complete, for the following basic reasons:

1. With a coefficient of excess oxidizer less than unity, i.e., when oxidizer is supplied to the combustion chamber in a quantity less than theoretically necessary, part of the combustible will not participate in the combustion at all.

2. As a result of the high combustion temperatures, the combustion of the

propellant will be accompanied by considerable dissociation, which leads to a still more incomplete combustion of the propellant.

3. As a result of the incomplete mixing of the components of the propellant a certain part of the combustible does not burn, in spite of the presence of free oxygen in the mixture.

Thus, at the exit of the combustion chamber and on entrance into the nozzle, the total energy of the combustion products will consist of the enthalpy whose value is determined by the temperature and the composition of the gas mixture; together with a very substantial amount of unutilized chemical energy. At high gas velocities at the exit of a high-speed combustion chamber, a certain amount of kinetic energy must also be taken into account.

The flow of the combustion products through the nozzle is likewise a rather complex process, namely: In addition to the conversion of part of the enthalpy into kinetic energy of efflux, there is a rather intense additional conversion of part of the residual chemical energy in the combustion products into heat, for the following reasons:

1. As a result of the considerable lowering of the gas temperature during expansion in the nozzle, a partial recombination of the molecules takes place;
2. There is an additional partial combustion of the combustible, which was not completed in the combustion chamber in view of the imperfect mixing.

These two processes represent the conversion, into thermal energy, of part of the residual chemical energy in the combustion products.

As the fundamental equation serving for the calculation of these processes, it is entirely expedient to apply the general energy equation ordinarily employed in the study of gas flow in gas dynamics, with a suitable modification of that equation.

This equation, for 1 kg of gas in the region of the duct 1 - 2, is of the general form:

$$\frac{Q}{A} + \left( \frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2} \right) = L + L_{fr} + (Z_2 - Z_1) + \frac{U_2 - U_1}{A} + \frac{w_2^2 - w_1^2}{2g}. \quad (26)$$

The following notations are used in this equation:

$Q$  is the heat supplied to the gas or removed from it, over the path 1 - 2;

$L, L_{fr}$  are the technical work and the work of friction in the segment of the duct;

$\frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2}$  is the work against the internal forces of pressure;

$U_2 - U_1$  is the change in the internal energy of the gas;

$\frac{w_2^2 - w_1^2}{2g}$  is the change in the external kinetic energy of the gas;

$Z_2 - Z_1$  is the change in the external potential energy of the gas.

In the case under study, no technical work is done, i.e.,  $L = 0$ , and the work of friction may be neglected in view of its smallness, i.e.,  $L_{fr} = 0$ . Neglecting the vanishingly small change in the energy of position of the gas, the energy equation for this case may be represented in the following simplified form:

$$\frac{Q}{A} + \left( \frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2} \right) = \frac{U_2 - U_1}{A} + \frac{w_2^2 - w_1^2}{2g}$$

or, after the usual transformations,

$$Q = i_2 - i_1 + A \frac{w_2^2 - w_1^2}{2g}. \quad (27)$$

In this equation all the heat participating in the process is taken as the value of  $Q$ : the heat supplied to the gas from outside the system or removed from it; the heat formed as a result of the conversion of chemical energy, i.e., due to the combustion process; the heat produced by friction, heat exchange with the surrounding space, etc. In this case, from the total quantity of heat we must separate the heat evolved on account of the conversion of chemical energy; let us represent this by the symbol  $x$ . Since heat is not supplied to the propellant nor to the

combustion products by other means, it follows that  $Q$  denotes only the heat of friction and the heat exchange with the surrounding space. Thus the general form of the energy equation will be as follows:

$$x_1 - x_2 = i_2 - i_1 + A \frac{w_2^2 - w_1^2}{2g} + Q \quad (28)$$

or

$$x_2 + i_2 + A \frac{w_2^2}{2g} = x_1 + i_1 + A \frac{w_1^2}{2g} - Q. \quad (29)$$

In view of the exceptionally great heat liberation per unit volume during the processes of combustion and discharge in a liquid-propellant rocket engine, the quantity  $Q$  is only an insignificant part of the total energy participating in the processes; this quantity may therefore be simply neglected. On the other hand, in practical calculations, these losses may be taken into account by selecting the proper values of the practical coefficients or by a suitable correction to the heat value of the propellant. Therefore, we will hereafter use the equation of energy in the form of eq.(28) or eq.(29), but without the quantity  $Q$ .

## 2. Coefficient of Heat Liberation

As stated above, the incomplete combustion of the propellant is due to three main causes. The incomplete combustion of the combustible, owing to the supply of an insufficient quantity of oxidizer, i.e., at  $\alpha < 1$ , is simple to calculate, if  $\alpha$  is specified. If  $\alpha = 0.8$ , this means that only 80% of the theoretically necessary quantity of oxygen is supplied, and consequently only 80% of all the combustible can be completely burned. To determine the proportion of combustible unburned for other causes, the proper coefficient must be introduced. In internal combustion piston engines, the coefficient of heat liberation  $\xi$  is adopted to allow for the quantity of heat liberated up to the moment the combustion products begin to expand. Such a coefficient is entirely acceptable for calculating the combustion process in a piston engine, where the combustion temperature is relatively low, since the at-

mospheric air is used as oxidizer and the degree of dissociation does not exceed 1.5 - 2%.

For calculating the combustion processes in a liquid-propellant rocket engine, the incomplete combustion of the combustible, due to the considerable dissociation of the gases, must be separately taken into account. For this reason the total coefficient of heat liberation  $\xi$  is to be regarded as the product of two coefficients:  $\xi_{inc}$ , a coefficient of heat liberation, which has a value less than unity as a result of the incomplete mixing of the fuel components,  $\xi_{diss}$ , a coefficient of heat liberation, which likewise has a value less than unity, owing to the dissociation of the combustion products.

Consequently,  $\xi = \xi_{inc} \cdot \xi_{diss}$ . The value of  $\xi_{diss}$  is found, as will be shown below, by solving the system of equations of chemical equilibrium; however, the value of  $\xi_{inc}$  can be derived only from practical data; this quantity depends on the propellant supply system in the liquid rocket engine, on the quantitative ratio of the propellant components, on their viscosity, on the temperature, and on a large number of other factors.

The insufficient state of our knowledge as to the value of the coefficient of heat liberation due to incomplete mixing,  $\xi_{inc}$ , makes it impossible to run thermal calculations with sufficient accuracy, allowing for the incomplete liberation of heat. For this reason, the calculations of the combustion process may be handled in two ways:

1. The incomplete combustion of the propellant, due to incomplete mixing of the components, may be included in the total energy losses in the combustion chamber, and the combustion may be calculated under the assumption that  $\xi_{inc} = 1$ .

2. After finding the practical value of  $\xi_{inc}$ , one may allow for it in the calculation by excluding these losses from the total losses.

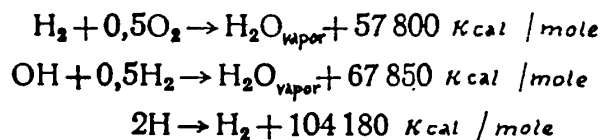
### 3. Chemical Energy of the Elements

To determine the chemical energy introduced by the propellant into the combus-



tion chamber, a method of estimating the chemical energy of the component elements must be established. There are apparently no generally adopted methods in the literature. Indeed, in the article "Calculations of Thermal Processes at High Temperatures", Ya.B.Zel'dovich and A.I.Polyarnyy take the chemical energies of  $O_2$ ,  $H_2$ ,  $CO_2$ , and  $H_2O$  as zero at  $0^\circ$  abs, while A.P.Vanichev, in his book: "Thermodynamic Calculation of Combustion and Efflux in the High-Temperature Region", takes the chemical energies of  $H_2$ ,  $O_2$ ,  $N_2$  and C as zero at  $293^\circ$  abs. This second method leads to calculations with negative values for the heat values of the propellants and the enthalpies of the combustion products.

In technical calculations, where the heat value is the primary quantity, it is more natural to adopt the former method of determining the chemical energy of the elements, but at a temperature of  $298^\circ$  abs adopted as the reference temperature. The determination of these quantities is based on the following procedure: The process of combustion of hydrogen, allowing for the possible dissociation, is expressed by the following chemical equations:



There are five gases in these three equations ( $H_2$ ,  $O_2$ ,  $H_2O_{\text{vapor}}$ , OH, and H). For two of these gases, we may take arbitrary values for their chemical energy, and to reduce the volume of numerical calculation, it is best to equate them to zero; in spite of the different numerical values for the different methods of selecting the values of the chemical energy, the final results of the calculations of the processes of combustion and efflux will undoubtedly be the same. The equation of energy balance for the above three chemical equations will have the form

$$\begin{aligned} X_{H_2} + 0,5X_{O_2} &= X_{H_2O \text{ vapor}} + 57\,800; \\ X_{OH} + 0,5X_{H_2} &= X_{H_2O \text{ vapor}} + 67\,850; \\ 2X_H &= X_{H_2} + 104\,180. \end{aligned}$$

For example, if we assume that  $X_{H_2} = 0$  and  $X_{O_2} = 0$  and solve the three equations, we shall find that  $X_{H_2O} = -57,800$  kcal/mole,  $X_{OH} = 10,050$  kcal/mole, and  $X_H = 52,090$  kcal/mole.

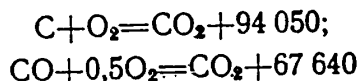
To obtain numerical values that are closest to those usually adopted in technical calculations, it is more expedient to take  $X_{O_2} = 0$  and  $X_{H_2O} = 0$ ; then the solution of the equation will give

$$X_H = 57\,800 \text{ kcal/mole}, \quad X_{OH} = 38\,950 \text{ kcal/mole}$$

and

$$X_H = 80\,990 \text{ kcal/mole}$$

From the two equations for the combustion of carbon:



at  $X_{O_2} = 0$  and at  $X_{CO_2} = 0$ , we find that  $X_C = 94,050$  kcal/mole and  $X_{CO} = 67,640$  kcal/mole.

All these calculations give the values for the chemical energy presented in Table 4\*.

Table 4

Substance	Chemical Formula	Chemical Energy in kcal/mole	Substance	Chemical Formula	Chemical Energy in kcal/mole
Hydrogen	H <sub>2</sub>	57 800	Steam	H <sub>2</sub> O	0
Carbon monoxide	CO	67 640	Monatomic hydrogen	H	80 990
Nitric oxide	NO	21 600	Monatomic oxygen	O	59 160
Hydroxyl	OH	38 950	Carbon	C	94 050
Oxygen	O <sub>2</sub>	0			
Carbon dioxide	CO <sub>2</sub>	0			

\*The values of the chemical energy are given in Table 4 for a reaction temperature of 25°C, and are rounded off to the nearest 10 kcal.

#### 4. Chemical Energy and Heat Value of a Propellant

As stated above, the concepts of chemical energy and heat value for individual elements, gases and propellant components coincide. For a propellant, however, these two concepts must be differentiated, namely: By the chemical energy of a propellant we mean all the chemical energy contributed by the propellant, regardless of whether it can be liberated in the form of heat during the combustion process or not, while the heat value of a propellant must be taken to mean only that part of the chemical energy which theoretically can be liberated in the form of heat if the combustion of the propellant is complete. Thus, at a coefficient of excess oxidizer  $\alpha$  equal to 0.8, the maximum that can be burned is only 0.8 of all the combustible supplied, but in calculating the chemical energy, the chemical energy of the entire quantity of combustible must be taken into account; if  $\alpha \geq 1$ , then the heat value of the propellant, after reaching a definite value at  $\alpha = 1$ , varies only slightly thereafter (just like the chemical energy) exclusively as a function of the heat value of the oxidizer, somewhat decreasing for a negative heat value of the oxidizer and increasing for a positive value. We present a more exact definition of the concept of the heat value of a propellant for a liquid rocket engine: The heat value of a propellant is the quantity of heat liberated on full utilization of one of the components during the combustion process.

In accordance with the above, the heat value of a propellant at  $\alpha < 1$  must be determined by the formulas

$$H_p = \alpha (H_{p_c} + k_o H_{p_o}) \text{ Kcal / mole} \quad (30)$$

or

$$H_p = \frac{\alpha (H_{p_c} + k_o H_{p_o})}{p_c + \alpha k_o \mu_o} \text{ Kcal / Kg.} \quad (31)$$

The chemical energy of a propellant will be determined by the formulas

$$X = H_{p_c} + \alpha k_o H_{p_o} \text{ Kcal / mole} \quad (32)$$

or

$$x = \frac{H_{\mu c} + \alpha k_0 H_{\mu o}}{\mu_c + \alpha k_0 \mu_o} \text{ Kcal / Kg.} \quad (33)$$

The heat value of a propellant for  $\alpha \geq 1$  is equal in value to the chemical energy and is determined by eqs.(32) and (33).

Figure 1 gives the above quantities for various values of  $\alpha$ , for the following two propellants:

1. Kerosene ( $H_c = 10,270$  kcal/kg, composition by weight  $C_c = 86\%$ ,  $H_c = 14\%$ ,

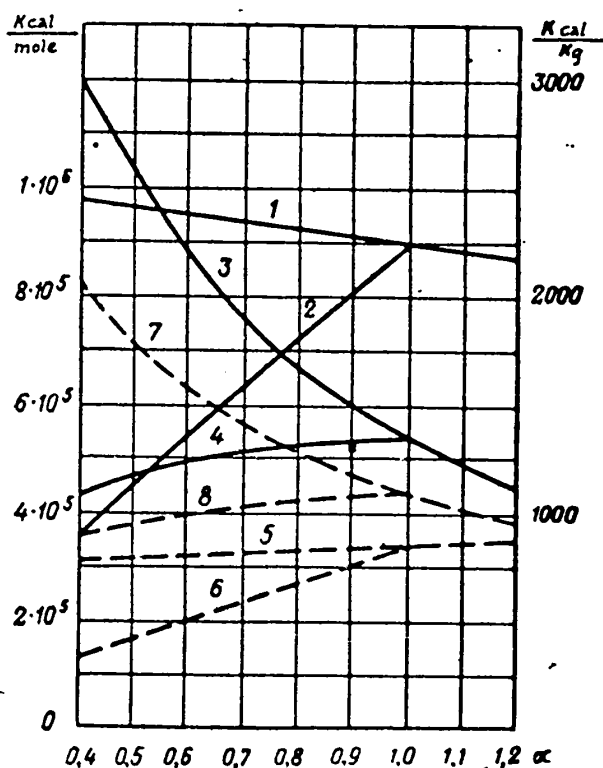


Fig.1 - Characteristic of Propellants

Kerosene + 95%  $\text{HNO}_3$

- 1 - Molecular chemical energy  $X_{\mu p}$ ; 2 - Molecular heat value  $H_{\mu p}$ ;  
3 - Chemical energy by weight  $X_p$ ; 4 - Heat value by weight  $H_p$

95%  $\text{C}_2\text{H}_6\text{O}$  + 80%  $\text{H}_2\text{O}_2$

- 5 - Molecular chemical energy  $X_{\mu p}$ ; 6 - Molecular heat value  $H_{\mu p}$ ;  
7 - Chemical energy by weight  $X_p$ ; 8 - Heat value by weight  $H_p$

arbitrary chemical formula  $C_{7.15}H_{14}$ ) and 95% nitric acid (chemical formula  $HNO_3 \cdot 0.184 H_2O$ , heat value  $H_{\mu o} = -14,905$  kcal/kg).

Consequently,

$$k_o = \frac{2.7,15 + 0.5 \cdot 14}{3 - 0.5 \cdot 1} = 8.5.$$

The formulas for calculation now take the following form:

$$X = 10\,270 \cdot 100 + 8.5 \cdot \alpha (-14\,905) = 1\,027\,000 - 126\,600\alpha$$

$$x = \frac{1\,027\,000 - 126\,600\alpha}{100 - 563.7\alpha};$$

For  $\alpha \leq 1$

$$H_{\mu} = (1\,027\,000 - 8.5 \cdot 14\,905) \alpha = 900\,400\alpha$$

$$H_s = \frac{900\,400\alpha}{100 - 563.7\alpha};$$

For  $\alpha \geq 1$

$$H_{\mu} = X \text{ and } H_s = x.$$

2. 95% ethyl alcohol (chemical formulas  $C_2H_6O \cdot 0.135 H_2O$ ,  $H_{\mu c} = 290,580$  kcal/kg,  $\mu_c = 48.4$ ) and 80% hydrogen peroxide (chemical formula  $H_2O_2 \cdot 0.472 H_2O$ ,  $H_{\mu} = 7240$  kcal/mole,  $\mu_o^* = 42.5$ );  $k_o = 6$ .

The corresponding computational formulas will have the form

$$\begin{aligned} X &= 290\,580 + 43\,440\alpha, & H_{\mu} &= 334\,020\alpha, \\ x &= \frac{290\,580 + 43\,440\alpha}{48.4 + 255\alpha}, & H_s &= \frac{334\,020\alpha}{48.4 + 255\alpha}. \end{aligned}$$

## CHAPTER III

## CALCULATIONS OF PROCESSES IN LIQUID-PROPELLANT ROCKET ENGINE

1. The Principal Processes in a Liquid-Propellant Rocket Engine

For the further calculations the following notation is adopted for the principal cross sections in liquid-propellant rocket engines (Fig.2).

At the section  $c - c$  the atomized mixture of the propellant components arrives, and the combustion process begins; in the section  $z - z$  the process of combustion is substantially concluded, and the combustion products enter the exhaust nozzle;

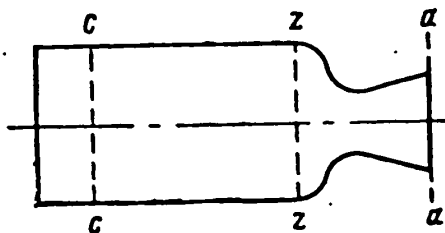


Fig.2 - For Calculating the Processes  
in a Liquid-Propellant Rocket Engine

the passage of the combustion products through the nozzle is accompanied by further combustion of the propellant, which has not been completed in the combustion chamber, and by a substantial recombination of the molecules, due to the decreased temperature during efflux.

The velocity of the chemical reaction of combustion of the propellant evidently varies for different rocket propellants, just as is the case for the duration of the preparatory period; besides the form of the propellant, a considerable number of other factors affect these quantities: the combustion temperature, the weight ratio of the components, the temperature of the arriving components, the degree of atomization and mixing, etc.

Thus the coefficient of heat liberation at the end of the combustion chamber,

$\xi_{inc z}$ , may have various values governed by the above factors. It is most natural that this coefficient should not be amenable to theoretical determination, meaning that it must be established on the basis of practical considerations and data for existing liquid-propellant rocket engines. In the same way, without data on the completeness of the afterburning of the propellant during the exhaust process, we must confine ourselves to practical and experimental data on the values of the coefficient of heat liberation in a cross section of the nozzle,  $\xi_{inc a}$ . The value of  $\xi_{inc a}$  also depends on the residence time of the combustion products in the nozzle, i.e., on the rate of flow of the gases and on the length of the nozzle.

The process of efflux is complicated not only by the afterburning of the propellant but also by the recombination of the molecules due to the drop in the temperature of the combustion products. There is no reliable assurance that the recombination of the molecules proceeds so rapidly that the combustion products are always in a state of chemical equilibrium, nor that the oscillatory energy of the atoms in the molecules (which is restored in any case more slowly than the rotational and translational energy of the molecules) may vary so rapidly that the combustion products are always in energetic equilibrium. There are no definite data as to how completely chemical and energetic equilibrium is restored. This is even more emphasized by the fact that these quantities also depend on the residence time of the combustion products in the nozzle and thus on the velocity of flow and on the length of the nozzle.

Theoretically, one might imagine two extreme hypotheses about the efflux of the combustion products:

1. Extreme non-equilibrium exhaust conditions, when it is assumed that the chemical and energetic equilibriums are restored very slowly; it is assumed in this case that the chemical composition of the combustion products does not vary at all, since there is no afterburning of the fuel, and the oscillatory energy of the atoms has remained constant. Thus, under this stipulation it is assumed that no additional

processes other than the conversion of part of the enthalpy of the combustion products into kinetic energy takes place, i.e., that in this case the process of efflux of the combustion products is adiabatic.

2. Complete equilibrium efflux, when it is assumed that both the chemical and energetic equilibriums are very rapidly restored; under such conditions it may be assumed that the chemical composition of the combustion products, like the oscillatory energy of the atoms, corresponds at all times to the temperature; moreover, there is also an additional partial afterburning of the propellant, so that the coefficient of heat liberation at emergence from the nozzle  $\xi_{inc a}$  is higher than the same coefficient at the exit of the combustion chamber,  $\xi_{inc z}$ . Thus, in this case, the process of efflux takes place with the addition of a considerable quantity of heat, due both to the afterburning of part of the propellant and to recombination of the molecules; consequently, this process must be regarded as a polytropic process with an index of polytropy  $n$  smaller than  $k$ .

The actual process of efflux must be somewhere between these two extremes, and the longer the nozzle and the longer the time of exhaust, the closer will the process come to complete equilibrium of efflux. Recent research has shown that the actual process of efflux takes place in a manner very close to the equilibrium state. It is therefore more rational to calculate precisely this equilibrium discharge, and then to apply practical coefficients to allow for the deviation of the process from equilibrium.

## 2. Composition of the Combustion Products

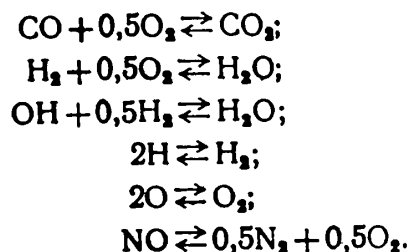
The principal quantities determining the composition of the combustion products are as follows: the coefficient of excess oxidizer  $\alpha$ , the coefficient of liberation of heat with respect to completeness of mixing  $\xi_{inc z}$ , and the actual combustion temperature  $T_z$ . The composition of the combustion products is very complex, owing to the considerable degree of dissociation. If we neglect the presence of



several gases contained in very small amounts in the combustion products, we may assume that in the general case the combustion products consist of  $\text{CO}_2$ ,  $\text{CO}$ ,  $\text{H}_2\text{O}$ ,  $\text{H}_2$ ,  $\text{OH}$ ,  $\text{N}_2$ ,  $\text{O}_2$ ,  $\text{NO}_2$ ,  $\text{H}$ , and  $\text{O}$ .

Calculations and experiments show that, at lower dissociation temperatures, there is more  $\text{H}_2$  than  $\text{OH}$ , while at higher temperatures the quantity of  $\text{OH}$  considerably exceeds that of  $\text{H}_2$ . The monatomic gases  $\text{H}$  and  $\text{O}$  appear in appreciable quantities only at temperatures over  $2400^\circ$  abs.

In the most generally adopted case, the dissociation of the combustion products is determined by the following chemical equations:



The quantity of gases in the combustion products is most conveniently determined from their partial pressures, since it is well known that

$$\frac{p_i}{p_z} = r_i, \quad (34)$$

i.e., the partial pressures determine the volumetric composition of the gas mixture.

In accordance with the above equations for the chemical reactions, the following six equations of chemical equilibrium may be set up:

$$\frac{p_{\text{CO}} \cdot p_{\text{O}_2}^{0,5}}{p_{\text{CO}_2}} = K_{p_1}; \quad (35)$$

$$\frac{p_{\text{H}_2} \cdot p_{\text{O}_2}^{0,5}}{p_{\text{H}_2\text{O}}} = K_{p_2}; \quad (36)$$

$$\frac{p_{\text{OH}} \cdot p_{\text{H}_2}^{0,5}}{p_{\text{H}_2\text{O}}} = K_{p_3}; \quad (37)$$

$$\frac{p_{H_2}}{p_{H_2}} = K_{p_1}; \quad (38)$$

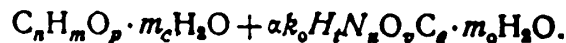
$$\frac{p_O^2}{p_{O_2}} = K_{p_2}; \quad (39)$$

$$\frac{p_{NO}}{p_{N_2}^{0.5} \cdot p_{O_2}^{0.5}} = K_{p_3}. \quad (40)$$

In these equations the equilibrium constants  $K_{p_1}$ ,  $K_{p_2}$  ... are taken from the appropriate Tables (Appendix I). The seventh equation determines the pressure of the combustion products as the sum of the partial pressures of the component gases, as follows:

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{OH} + p_{N_2} + p_{O_2} + p_{NO} + p_{H} + p_{O} = p_r. \quad (41)$$

Three more equations must be set up to permit determining the composition of the combustion products at a specified temperature. These equations are set up as equations of material balance, based on the data on the composition of the fuel components and on their molar ratio. In the most general case, the mixture of the components before combustion is defined by the sum:



This mixture contains the following number of atoms of the component elements:

Carbon	.....	$n + a k_o q$
Hydrogen	.....	$m + 2m_c + a k_o (t + 2m_o)$
Oxygen	.....	$p + m_c + a k_o (v + m_o)$
Nitrogen	.....	$a k_o u$

This number of atoms must persist in the combustion products formed. Carbon is contained in only two gases,  $CO_2$  and  $CO$ , one atom to the molecule of each gas; consequently,

$$M_{CO_2} + M_{CO} = n + a k_o q.$$

Hydrogen is contained in hydroxyl  $OH$  and in monatomic hydrogen, one atom in each, while water vapor  $H_2O$  and diatomic hydrogen contain two atoms each; consequently,

$$2(M_{H_2O} + M_{H_2}) + M_{OH} + M_H = m + 2m_c + \alpha k_o (t + 2m_o).$$

For oxygen, we get

$$2(M_{CO_2} + M_{O_2}) + M_{CO} + M_{H_2O} + M_{OH} + M_{NO} + M_O = p + m + \alpha k_o (v + m_o),$$

and for nitrogen,

$$2M_{N_2} + M_{NO} = \alpha k_o u.$$

The unknowns in these four equations are the numbers of moles in the gases:

$M_{CO_2}$ ,  $M_{CO}$ ,  $M_{H_2O}$ , etc.

They may be replaced by expressions in terms of the partial pressures; the equation for any component gas will have the form

$$p_i = r_i p_s = \frac{M_i}{M} p_s$$

or

$$M_i = M \frac{p_i}{p_s}.$$

Let us rewrite the equation so obtained in the following way:

$$\begin{aligned} \frac{M}{p_s} (p_{CO_2} + p_{CO}) &= m + \alpha k_o q, \\ \frac{M}{p_s} [2(p_{H_2O} + p_{H_2}) + p_{OH} + p_H] &= m + 2m_c + \alpha k_o (t + 2m_o); \\ \frac{M}{p_s} [2(p_{CO_2} + p_{O_2}) + p_{CO} + p_{H_2O} + p_{OH} + p_{NO} + p_O] &= p + m_c + \alpha k_o (v + m_o), \\ \frac{M}{p_s} (2p_{N_2} + p_{NO}) &= \alpha k_o u. \end{aligned}$$

These four equations contain the new unknown  $M$ , the total number of moles of combustion products. This unknown may be eliminated by dividing any three of these equations by the fourth one; it is most convenient to divide the three last equations by the first one.

After division, we get the three missing equations of material balance:

$$\frac{2(p_{H_2O} + p_{H_2}) + p_{OH} + p_H}{p_{CO_2} + p_{CO}} = \frac{m + 2m_c + \alpha k_o (t + 2m_o)}{n + \alpha k_o q}, \quad (42)$$

$$\frac{2(p_{\text{CO}_2} + p_{\text{O}_2}) + p_{\text{CO}} + p_{\text{H}_2\text{O}} + p_{\text{OH}} + p_{\text{NO}} + p_{\text{O}}}{p_{\text{CO}_2} + p_{\text{CO}}} = \frac{p + m_c + ak_o(v + m_o)}{n + ak_oq}, \quad (43)$$

$$\frac{2p_{\text{N}_2} + p_{\text{NO}}}{p_{\text{CO}_2} + p_{\text{CO}}} = \frac{ak_o u}{n + ak_oq}. \quad (44)$$

If the fuel components are specified, not by the chemical formula but by the elementary composition by weight, then the equations of material balance are set up on the basis of the following considerations: The weight of oxygen in 1 kg of fuel at an oxygen concentration  $\sigma_c\%$  is equal to

$$\frac{O_c}{100} \cdot \frac{\sigma_c}{100} \text{ kg},$$

while, in the corresponding quantity of oxidizer, its weight is  $\frac{ak'_o O_o}{100} \cdot \frac{\sigma_o}{100} \text{ kg}$ ; moreover, the weight of oxygen in the water is

$$\frac{8}{9 \cdot 100} [100 - \sigma_c + ak'_o (100 - \sigma_o)] \text{ kg}$$

and calculations of the weight of hydrogen yield:

$$\text{In the combustible} \quad \frac{H_c}{100} \cdot \frac{\sigma_c}{100} \text{ kg},$$

$$\text{In the oxidizer} \quad \frac{ak_o H_o}{100} \cdot \frac{\sigma_o}{100} \text{ kg},$$

$$\text{In water} \quad \frac{1}{9 \cdot 100} [100 - \sigma_c + ak'_o (100 - \sigma_o)] \text{ kg};$$

For carbon:

$$\text{In the combustible} \quad \frac{C_c}{100} \cdot \frac{\sigma_c}{100} \text{ kg},$$

$$\text{In the oxidizer} \quad \frac{ak'_o C_o}{100} \cdot \frac{\sigma_o}{100} \text{ kg}.$$

Simple arguments lead to the following equations:

$$\begin{aligned} & \frac{p + m_c + ak_o(v + m_o)}{n + ak_oq} = \\ & = \frac{12}{16} \cdot \frac{O_c \sigma_c + ak'_o O_o \sigma_o + 100 \cdot \frac{8}{9} [100 - \sigma_c + ak'_o (100 - \sigma_o)]}{C_c \sigma_c + ak'_o C_o \sigma_o}, \\ & \frac{m + 2m_c + ak_o(t + 2m_o)}{n + ak_oq} = \end{aligned}$$

$$= \frac{12}{1} \cdot \frac{H_c \sigma_c + a k'_0 H_o \sigma_o + 100 \cdot \frac{1}{9} [100 - \sigma_c + a k'_0 (100 - \sigma_o)]}{C_c \sigma_c + a k'_0 C_o \sigma_o},$$

$$\frac{a k'_0 u}{n + a k'_0 q} = \frac{12}{14} \cdot \frac{a k'_0 N_o \sigma_o}{C_c \sigma_c + a k'_0 C_o \sigma_o}.$$

Consequently, the equations of material balance in this case are brought into the form of

$$\begin{aligned} & \frac{2p_{CO_2} + p_{CO} + p_{H_2O} + p_{OH} + 2p_O + p_{NO} + p_O}{p_{CO_2} + p_{CO}} = \\ & = 0,75 \frac{O_c \sigma_c + a k'_0 O_o \sigma_o + 100 \cdot \frac{8}{9} [100 - \sigma_c + a k'_0 (100 - \sigma_o)]}{C_c \sigma_c + a k'_0 C_o \sigma_o}; \\ & \frac{2p_{H_2O} + 2p_{H_2} + p_{OH} + p_H}{p_{CO_2} + p_{CO}} = \\ & = 12 \frac{H_c \sigma_c + a k'_0 H_o \sigma_o + 100 \cdot \frac{1}{9} [100 - \sigma_c + a k'_0 (100 - \sigma_o)]}{C_c \sigma_c + a k'_0 C_o \sigma_o}; \quad (45) \\ & \frac{2p_{N_2} + p_{NO}}{p_{CO_2} + p_{CO}} = \frac{6}{7} \cdot \frac{a k'_0 N_o \sigma_o}{C_c \sigma_c + a k'_0 C_o \sigma_o}. \end{aligned}$$

The above ten equations permit determining the composition of the combustion products for any desired temperature. In calculating the combustion process, the composition of the combustion product is determined for two or three selected temperatures, after which, using the equation of energy, the actual combustion temperature is found and thus also the actual composition of the combustion products.

### 3. The Equation of Energy in the Combustion Process

The equation of energy, allowing for the incomplete liberation of heat as a result of incomplete atomization and mixing of the components, is constructed on the basis of eq.(28) or eq.(29). With total heat liberation, i.e., for  $\xi_{inc} = 1$ , eq.(28) takes the form

$$x_c - x_s = i_s - i_c + A \frac{w_s^2 - w_c^2}{2g} + Q.$$

Neglecting the quantity  $Q$  and the variation in kinetic energy, i.e., assuming that

$$Q=0, \\ A \frac{w_z^2 - w_c^2}{2g} = 0,$$

which is entirely permissible for a low-speed combustion chamber, eq.(28) may be represented in the form

$$x_c - x_z = i_z - i_c$$

or

$$x_z + i_z = x_c + i_c.$$

Hereafter, for simplicity, we shall introduce the term energy content for the sum of the chemical and thermal energy, and shall denote it by  $e$ .

Thus the general equation of energy for the combustion process may be represented in the form

$$e_z = e_c,$$

where

$$e_z = x_z + i_z,$$

$$e_c = x_c + i_c.$$

If the coefficient of heat liberation is less than unity, then, instead of the quantity of heat that could be liberated on total combustion, namely  $H_p$ , only  $\xi_{inc} H_p$  will be liberated, while a part of the heat, amounting to  $(1 - \xi_{inc}) H_p$  remains in the form of chemical energy and must be taken into account in eq.(28) by means of the separate term. Thus, the equation for the combustion process, allowing for incomplete combustion, will have the form

$$x_z + i_z + (1 - \xi_{inc}) H_p = x_c + i_c$$

or

$$x_z + i_z = x_c + i_c - (1 - \xi_{inc}) H_p. \quad (46)$$

Applying the concept of energy content, we get

$$e_z = e_c - (1 - \xi_{inc}) H_p,$$

where all the quantities on the right side of the equation are known, if the value of the coefficient of heat liberation  $\xi_{inc}$  is selected on the basis of practical

considerations.

On the left side of the equation, the quantities  $x_z$  and  $t_z$  represent the chemical energy and enthalpy of 1 kg of combustion products at the exit of the combustion chamber. These quantities, on the basis of the solution of a system of the ten equations given above, may be expressed as follows: For any  $i^{\text{th}}$  gas we may write

$$M_i = r_i M = \frac{p_i}{p_z} M,$$

where  $M$  is the total number of moles of combustion products, and  $M_i$  the number of moles of the  $i^{\text{th}}$  gas. Knowing the number of moles of each gas in the combustion products, and having the tabular data of the values of  $I_i$  for the temperature selected as well as for the values of all  $x_i$ , we may write:

$$\left. \begin{aligned} I_z &= \sum M_i I_i = \frac{M}{p_z} \sum p_{iz} I_{iz} \\ X_z &= \sum M_i X_i = \frac{M}{p_z} \sum p_{iz} X_i \end{aligned} \right\} \quad (47)$$

In order to recalculate eqs. (47) for 1 kg of combustion products, they must be divided by the weight of the combustion products obtained on the combustion of 1 mole of combustible; this weight, at  $\sigma_c\%$  concentration of the combustible and at  $\sigma_o\%$  concentration of the oxidizer, is equal to

$$G_p = \mu_c + \alpha k_o \mu_o + 18(m_c + \alpha k_o m_o) = 100 \left( \frac{\mu_c}{\sigma_c} + \frac{\alpha k_o \mu_o}{\sigma_o} \right).$$

On the other hand, the molecular weight of the combustion products is

$$\mu_z = \frac{G_p}{M}$$

and, consequently,

$$G_p = M \mu_z.$$

Hence, making use of eq. (47), we may write

$$i_z = \frac{I_z}{G_p} = \frac{1}{\mu_z p_z} \sum p_{iz} I_{iz} \quad (48)$$

$$x_z = \frac{X_z}{G_p} = \frac{1}{\mu_z p_z} \sum p_{iz} X_i \quad (49)$$

Thus the left side of eq.(46), if eqs.(48) and (49) are used, is brought into the form:

$$i_z + x_z = \frac{1}{\mu_z p_z} \left( \sum p_{iz} I_{iz} + \sum p_{iz} X_i \right) = \frac{1}{\mu_z p_z} \sum p_{iz} (I_{iz} + X_i)$$

or

$$e_z = \frac{1}{\mu_z p_z} \sum p_{iz} E_{iz}. \quad (50)$$

The values of  $E_{iz}$  for various gases at various temperatures are given in Appendix II.

The molecular weight of the combustion products is determined from the formula

$$\mu_z = \sum r_i \mu_i = \sum \frac{p_{iz}}{p_z} \mu_i = \frac{1}{p_z} \sum p_{iz} M_i. \quad (51)$$

Consequently,

$$\left. \begin{aligned} i_z &= \frac{\sum p_{iz} I_{iz}}{\sum p_{iz} \mu_i}, \\ x_z &= \frac{\sum p_{iz} X_i}{\sum p_{iz} \mu_i}, \end{aligned} \right\} \quad (52)$$

$$e_z = \frac{\sum p_{iz} E_{iz}}{\sum p_{iz} \mu_i}. \quad (53)$$

Thus for any arbitrarily selected temperature, all calculations may be performed to determine both the composition of the combustion products and their enthalpy, together with the amount of chemical energy remaining unutilized.

For the three arbitrarily selected temperatures  $T_z'$ ,  $T_z''$  and  $T_z'''$ , let  $x_z'$ ,  $i_z'$ ,  $x_z''$ ,  $i_z''$ ,  $x_z'''$ , and  $i_z'''$ , respectively, be found. The actual combustion temperature may be determined by interpolation, using the following three equations:

$$\begin{aligned} A + BT_z' + CT_z'^2 &= x_z' + i_z', \\ A + BT_z'' + CT_z''^2 &= x_z'' + i_z'', \\ A + BT_z''' + CT_z'''^2 &= x_z''' + i_z'''. \end{aligned}$$

From these equations, we determine the values of the coefficients A, B, and C and then set up the interpolation equation:

$$A + BT_z + CT_z^2 = e_z = e_c - (1 - \xi_{\mu_z}) H, \quad (54)$$



from which the actual combustion temperature  $T_z$  is determined. To determine the actual composition of the combustion products, partial pressure curves may be plotted in the axes  $p_i$  vs.  $T_z$  (Bibl.1) and from them the partial pressures at a temperature  $T_z$  may be found. For practical calculations, in most cases, the curvilinear temperature dependence may be replaced by a linear dependence, which yields sufficient accuracy if the temperature range is not particularly large.

In that case the interpolation equation takes the form:

$$M + NT_z = x_c + i_c - (1 - \xi_{ex}) H_p, \quad (55)$$

To determine the coefficients M and N, the following two equations are set up:

$$M + NT'_z = x'_z + i'_z,$$

$$M + NT''_z = x''_z + i''_z,$$

whence

$$N = \frac{x'_z + i'_z - (x''_z + i''_z)}{T'_z - T''_z},$$

$$M = \frac{(x'_z + i'_z) T''_z - (x''_z + i''_z) T'_z}{T'_z - T''_z}.$$

The solution of eq.(55) gives

$$T_z = \frac{x_c - (1 - \xi_{ex}) H_p - M}{N}. \quad (56)$$

#### 4. The Equation of Energy in the Exhaust Process

In the general case, as stated above, the equation of energy for calculating the processes of combustion and exhaust has the form of eq.(29):

$$x_2 + i_2 + A \frac{w_2^2}{2g} = x_1 + i_1 + A \frac{w_1^2}{2g}.$$

In calculating the process of efflux of the combustion products through the exhaust nozzle, the right side of the equation represents the total quantity of energy in the combustion products at the beginning of the efflux, i.e., at the exit

of the combustion chamber, namely

$$x_s + i_s + (1 - \xi_{inc}) H_p + A \frac{w_s^2}{2g}$$

or

$$e_s + (1 - \xi_{inc}) H_p + A \frac{w_s^2}{2g},$$

while for low-speed combustion chambers the last term of this expression may be neglected. The left side of the equation must represent the total quantity of energy in the combustion products on emergence from the nozzle, i.e., in the cross section a - a (see Fig.2). This energy consists of the remaining unutilized chemical energy  $x_a$  which, owing to dissociation, was not transformed into heat, the enthalpy of the combustion products  $i_a$ , the kinetic energy  $A \frac{w_a^2}{2g}$  and the chemical energy which, as a result of the incomplete combustion of the propellant, was not liberated in the form of heat  $(1 - \xi_{inc a}) H_p$ . Under such conditions, the general equation of energy for the process of efflux takes the form

$$x_a + i_a + A \frac{w_a^2}{2g} + (1 - \xi_{inc a}) H_p = x_s + i_s + (1 - \xi_{inc}) H_p + A \frac{w_s^2}{2g}.$$

For nonvelocity combustion chambers, as already stated, the last term of the right side may be neglected without appreciable error. In this case, the equation takes the form

$$x_a + i_a + A \frac{w_a^2}{2g} + (1 - \xi_{inc a}) H_p = x_s + i_s + (1 - \xi_{inc}) H_p \quad (57)$$

or, since

$$x_s + i_s = x_c + i_c - (1 - \xi_{inc}) H_p$$

we may write finally

$$x_a + i_a + A \frac{w_a^2}{2g} = x_c + i_c - (1 - \xi_{inc a}) H_p \quad (58)$$

or

$$e_a + A \frac{w_a^2}{2g} = e_c - (1 - \xi_{inc a}) H_p.$$

This equation, in combination with the ten equations given above, makes it

possible to perform all necessary calculations. In the following Sections we shall give the methods of calculating the process of efflux under the two extreme assumptions set forth above.

##### 5. Calculation of Extreme Case of Nonequilibrium Efflux

The assumption of the extreme case of nonequilibrium efflux includes the following assertions:

1. No recombination of molecules takes place during efflux, the dissociated combustion product remains dissociated, in spite of the considerable temperature drop during efflux (extreme chemical nonequilibrium).

2. The heat capacities of the combustion products, and consequently also the value of the adiabatic index  $k$ , maintain values determined only by the rotational and translational motion of the molecules; the oscillatory energy of the atoms remains unchanged.

3. The process of afterburning of the fuel during efflux of the combustion product does not occur, i.e.,  $\xi_{\text{inc } a} = \xi_{\text{inc } z}$ .

It follows that  $x_a = x_z$  and  $\xi_a = \xi_z$ , and the efflux is characterized by a single process, the conversion of part of the enthalpy of the combustion product into kinetic exhaust energy, i.e., in this case, the exhaust process will be adiabatic. Under these conditions, eq.(57) takes the form

$$i_a + A \frac{w_a^2}{2g} = i_z,$$

whence

$$w_a = \sqrt{\frac{2g}{A} (i_z - i_a)} = 91.53 \sqrt{i_z - i_a}. \quad (59)$$

In this equation, the enthalpy of the combustion products at the emergence from the nozzle is the unknown quantity; it may be determined from the equation

$$i_a = \frac{T_a}{p_a} \sum p_{ia} \mu C_{pi} \quad (60)$$

where the values of the  $\mu C_{pi}$  are taken from the data for molecular heat capacities given below.

For determining the values of  $I_{ia}$  at a known temperature, the Table of Appendix II may be used; from the value of  $E_i = I_i + X_i$  found from the Table we subtract the value of the chemical energy  $X_i$ , taken either from the same Table at  $T_a = 298^\circ$  abs or from Table 4 (in the text). It should be remembered that, in the system of reference adopted, oxygen, nitrogen, water vapor, and carbon dioxide possess no chemical energy, and that for them  $I_i = E_i$ .

To determine the values of  $I_{ia}$  it is necessary to know the temperature  $T_a$  of the gases on emergence from the nozzle; this temperature is determined from the usual relation for an adiabatic process:

$$\frac{T_a}{T_s} = \left( \frac{p_a}{p_s} \right)^{\frac{k-1}{k}}. \quad (61)$$

To determine the values of the heat capacities  $\mu C_p$  and  $\mu C_v$ , and also the value of  $k$ , one must start out from the following considerations.

At the initial moment of expansion of the combustion products, they are in chemical and energetic equilibrium. Energetic equilibrium assumes that the oscillatory motions of the atoms proceed in accordance with the temperature to which the gases were heated during the combustion process. On expansion of the gases in the nozzle, in accordance with the assumption of energetic nonequilibrium of the process that we have adopted, there are no variations in the oscillatory energy; consequently, the heat contents of the gases are determined only by the translational and rotational motion of the molecules, and are independent of the temperature variations.

These heat capacities may be determined from the values of the molar heat capacities calculated according to the molecular-kinetic theory of heat capacities; we may take, in round numbers:

For monatomic gases:

$$\mu C_v = 2.98 \text{ kcal /mole ;}$$

$$\mu C_p = 2,98 + 1,99 = 4,97 \text{ Kcal / mole};$$

For diatomic gases:

$$\mu C_v = 4,96 \text{ Kcal / mole}; \quad \mu C_p = 4,96 + 1,99 = 6,95 \text{ Kcal / mole}$$

For triatomic gases one must use the corrected values of the heat capacity:

$$\begin{array}{l} \text{for CO}_2 \quad \mu C_v = 6,71 \text{ Kcal / mole}; \quad \mu C_p = 8,70 \text{ Kcal / mole}; \\ \text{for H}_2\text{O} \quad \mu C_v = 6,80 \text{ Kcal / mole}; \quad \mu C_p = 8,79 \text{ Kcal / mole}. \end{array}$$

The heat capacities of the gas mixture are determined by the formulas

$$\begin{aligned} \mu C_p &= \frac{1}{p_z} \sum p_{iz} \mu C_{pi} \\ \mu C_v &= \frac{1}{p_z} \sum p_{iz} \mu C_{vi} \end{aligned} \quad (62)$$

#### 6. Calculation of Extreme Equilibrium Efflux

Under equilibrium efflux, the combustion products are in chemical and energetic equilibrium in all cross sections of the nozzle and, consequently:

1. The composition of the combustion products varies at all times, as a result of the recombination of molecules due to the temperature drop during exhaust.
2. The oscillatory energy of the atoms varies with the decreasing temperature, so that also the values of the heat capacity  $c_p$  and  $c_v$  decrease, while  $k$  increases.
3. To supplement the above conditions, we may introduce the assumption that the fuel, during the exhaust process, has time to undergo partial afterburning, as a result of which the coefficients of heat liberation must increase ( $\epsilon_{\text{inc a}} > \epsilon_{\text{inc z}}$ ).

The first and third conditions established the conversion into heat, during discharge, of a considerable part of the chemical energy contained in the combustion product before entering the nozzle; consequently, the exhaust process takes place under addition of heat to the combustion products, i.e., the exhaust process in this case must be regarded as polytropic, with an index of polytropy  $n$  less than  $k$ .

At the same time, the introduction of the second condition assumes the tempera-

ture dependence of the heat capacities, and consequently, the increase of the adiabatic index with decreasing temperature.

All the above factors determine the method of calculating the equilibrium efflux of the combustion products. The fundamental equation for the calculation is eq.(58), which has the form

$$e_a + A \frac{w_a^2}{2g} = e_c,$$

if the calculation is performed under the assumption of ideal mixing of the components ( $\xi_{inc a} = \xi_{inc z} = 1$ ) or

$$e_a + A \frac{w_a^2}{2g} = e_c - (1 - \xi_{inc a}) H_p,$$

if it is desired to make allowance in the calculation for the non-afterburning of the fuel as a result of incomplete mixing.

In this equation the right side is known if the value of the coefficient of heat liberation in the cross section of the nozzle  $\xi_{inc a}$  is first established on the basis of practical data.

The solution of this equation, as in the case of calculation of the combustion process, reduces down to a selection of the temperature  $T_a$  at which eq.(58) becomes an identity.

For any predetermined temperature,  $T_a$  may be found by using the Table from Appendix II:

$$e_a = \frac{1}{p_a v_a} \sum p_{ia} E_{ia}.$$

The value of the second term on the left side of the equation is determined from the conventional equation

$$A \frac{w_a^2}{2g} = A \frac{n}{n-1} RT_z \left[ 1 - \left( \frac{p_a}{p_z} \right)^{\frac{n-1}{n}} \right], \quad (63)$$

where

$$n = \frac{\lg \frac{p_2}{p_a}}{\lg \frac{p_2}{p_a} \frac{T_a}{T_2}} \quad (64)$$

To determine the quantities  $i_a$ ,  $x_a$ , and  $e_a$  at the selected temperature  $T_a$ , the above system of equations (35) to (44) must be solved. The solution of this system in this case is sometimes considerably simplified as a result of the cooling of the gases during efflux. Calculations show that for a kerosene + nitric acid propellant, at the temperatures of emergence from the nozzle, the monatomic gases H and O are entirely absent; while hydroxyl OH, nitric oxide NO, and oxygen  $O_2$  are present in such small quantities that they may be neglected and the products of combustion assumed to consist only of the following gases:  $CO_2$ , CO,  $H_2O$ ,  $H_2$ ,  $N_2$ . Under such conditions, eqs.(37), (38), (39), and (40) are dropped, and from eqs.(35) and (36) an equation for eliminating  $p_{O_2}$  from them is set up.

Thus the computational system of equations will consist of the following five:

$$\frac{p_{CO} p_{H_2O}}{p_{CO_2} p_{H_2}} = \frac{K_{p1}}{K_{p2}} \quad (65)$$

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = p_a \quad (66)$$

$$\frac{2(p_{H_2O} + p_{H_2})}{p_{CO_2} + p_{CO}} = \frac{m + 2m_s + ak_o(t + 2m_o)}{n + ak_oq} \quad (67)$$

$$\frac{2p_{CO_2} + p_{CO} + p_{H_2O}}{p_{CO_2} + p_{CO}} = \frac{p + m_c + ak_o(v + m_o)}{n + ak_oq} \quad (68)$$

$$\frac{2p_{N_2}}{p_{CO_2} + p_{CO}} = \frac{ak_o x}{n + ak_oq} \quad (69)$$

For any arbitrary selected temperature  $T_a$ , in this system of equations, only the value of  $\frac{K_{p1}}{K_{p2}}$  will vary. The solution of this system reduces to a determination of the value  $p_{CO_2}$  from a quadratic equation in which only one root has a physically real value, since the other root is negative. The solution of this system of equations will be shown in the examples of calculation given below. For propellants where the oxidizer is oxygen, such a simplification is not recommended, since it leads to considerable inaccuracies.

It must be borne in mind that, owing to the recombination of molecules and the partial additional afterburning of the fuel, the composition of the combustion products will vary substantially, and their molecular weight will increase somewhat, leading to a decrease in the gas constants of the mixture. In most cases these changes are small and may be neglected, but, at higher combustion temperatures when strong molecular recombination takes place during the exhaust process, the variations in molecular weight and gas constants become substantial; in such cases the exhaust process must be assumed as occurring at a mean value of the gas constant.

Thus, on determining the molecular weight of the combustion products on emergence from the nozzle by a formula analogous to eq.(51), namely

$$\mu_a = \frac{1}{p_a} \sum p_{ia} \mu_i \quad (70)$$

we get

$$\frac{\mu_z + \mu_a}{2} = \mu \text{ and } R = \frac{848}{\mu}. \quad (71)$$

The velocity of exhaust of the combustion products may then be determined by the formula

$$w_a = \sqrt{2g \frac{n}{n-1} RT_z \left[ 1 - \left( \frac{p_a}{p_z} \right)^{\frac{n-1}{n}} \right]}. \quad (72)$$

Thus we have found all the data for calculating the nozzle.



## CHAPTER IV

## EXAMPLES OF CALCULATIONS

Example 5. Calculate the processes of combustion and efflux of the combustion products for the propellants: tractor kerosene ( $\gamma_c = 0.82$  kg/ltr,  $C_c = 86.3\%$ ,  $H_c = 13.7\%$ ,  $H_u = 10,275$  kcal/kg) and 95% nitric acid ( $\gamma_o = 1.5$  kg/ltr,  $m_o = 0.184$  mole/mole). The coefficient of excess oxidizer is taken as  $\alpha = 0.8$ ; the coefficient of heat liberation at the exit of the combustion chamber, as  $\xi_{inc z} = 0.92$ ; in calculating the extreme equilibrium efflux we take  $\xi_{inc a} = 0.98$ . The pressure in the combustion chamber is assumed as  $p_z = 25$  kg/cm<sup>2</sup>, and the pressure at emergence from the nozzle as  $p_a = 1$  kg/cm<sup>2</sup>.

The arbitrary chemical formula for kerosene has the form



For a specified coefficient of excess oxidizer, the molar ratio of the propellant components will be

$$\alpha k_o = 0.8 \frac{2 \cdot 7.19 + 0.5 \cdot 13.7}{3 - 0.5} = 6.8 \text{ mole / mole}$$

With an arbitrary molecular weight of kerosene, taken as  $\mu_c = 100$ , its molar heat value will be

$$H_{\mu} = 100 \cdot 10275 = 1027500 \text{ kcal / mole}$$

For 95% nitric acid, eq.(8) gives

$$H_{\mu_o} = 57800 \cdot 0.5 \cdot 1 - 41400 - 10500 \cdot 0.184 = -14430 \text{ kcal / mole}$$

The weight coefficient of the component ratio, by eq.(12), will be:

$$\alpha k'_0 = 6,8 \frac{63}{100} \frac{100}{95} = 4,51 \text{ kg/kg.}$$

The volume coefficient of the component ratio, according to eq.(13) will be

$$\alpha k''_0 = 4,51 \frac{0,82}{1,5} = 2,465 \text{ ltr/ltr}$$

The weight of 1 mole of 95% nitric acid is

$$\mu'_0 = 63 \frac{100}{95} = 66,32 \text{ kg/mole}$$

The heat value of the propellant used is determined from eqs.(30) and (31) to be

$$H_\mu = 0,8 [1027500 + 8,5 (-14430)] = 723880 \text{ kcal /mole}$$

or

$$H_\mu = \frac{723880}{100 + 6,8 \cdot 66,32} = 1314 \text{ kcal /kg.}$$

The chemical energy of the propellant is determined from eqs.(32) and (33):

$$X = 1027500 + 6,8 (-14430) = 929380 \text{ kcal /mole}$$

or

$$x = \frac{929380}{100 + 6,8 \cdot 66,32} = 1673 \text{ kcal /kg}$$

Thus 1 kg of this propellant introduces 1673 kcal of chemical energy into the combustion chamber, but on complete combustion the maximum heat that can be liberated is only 1314 kcal, since the oxygen will be insufficient for complete combustion of the combustible, since only 80% of the theoretically necessary oxidizer have been supplied.

#### 1. Calculation of the Combustion Process

The combustion process may be calculated under two assumptions:

- a) The incompleteness of the combustion of the propellants due to incomplete mixing relates to the total losses of energy in the combustion chamber, taken into

account by the appropriate value for the efficiency of the combustion chamber.

Under this assumption, the coefficient of heat liberation due to incomplete mixing takes a value equal to unity, i.e.,  $\xi_{inc z} = 1$ .

b) Incomplete combustion of the propellant as a result of incomplete mixing is evaluated in practice by an appropriate choice of the value of the coefficient of heat liberation  $\xi_{inc z} < 1$ ; in this case  $\xi_{inc z} = 0.92$ .

$$a. \xi_{inc z} = 1$$

Equation (46) takes the form

$$x_s + i_s = x_c + i_c$$

or

$$e_s = e_c.$$

The initial enthalpy of the liquid fuel is determined from the equation:

$$i_c = \frac{(C_c p_c + a k_o C_{a'o}) t_c}{\mu_c + a k_o \mu_o} = \frac{(0,632 \cdot 100 + 6,8 \cdot 0,481 \cdot 66,32) 25}{100 + 6,8 \cdot 66,32} = 13 \text{ kcal / kg.}$$

We found above that

$$x_c = 1673 \text{ kcal / kg,}$$

Consequently,

$$e_c = 1673 + 13 = 1686 \text{ kcal / kg,}$$

and the equation of energy for the combustion products takes the form

$$e_s = i_s + x_s = 1686 \text{ kcal / kg.}$$

The system of equations for determining the composition of the combustion products at a predetermined temperature, is obtained in the following way:

The right sides of the equations of material balance (42), (43), and (44) take the following numerical values in this case.

For eq. (42):

$$\frac{m + 2m_c + a k_o (t + 2m_o)}{n + a k_o q} = \frac{13,7 + 2 \cdot 0 + 6,8 (1 + 2 \cdot 0,184)}{7,19 + 6,8 \cdot 0} = 3,2.$$

For eq. (43):

$$\frac{p + m_c + a k_o (v + m_o)}{n + a k_o q} = \frac{0 + 0 + 6,8 (3 + 0,184)}{7,19 + 6,8 \cdot 0} = 3,01,$$

for eq.(44):

$$\frac{ak_{ou}}{n + ak_{og}} = \frac{6,8 \cdot 1}{7,19 + 6,8 \cdot 0} = 0,945.$$

Thus the system of equations is brought into the form:

$$\frac{p_{CO} \cdot p_{O_2}^{0,5}}{p_{CO_2}} = K_{p_1}; \quad (a)$$

$$\frac{p_{H_2} \cdot p_{O_2}^{0,5}}{p_{H_2O}} = K_{p_2}; \quad (b)$$

$$\frac{p_{OH} \cdot p_{H_2}^{0,5}}{p_{H_2O}} = K_{p_3}; \quad (c)$$

$$\frac{p_H^2}{p_{H_2}} = K_{p_4}; \quad (d)$$

$$\frac{p_O^2}{p_{O_2}} = K_{p_5}; \quad (e)$$

$$\frac{p_{NO}}{p_{N_2}^{0,5} \cdot p_{O_2}^{0,5}} = K_{p_6}; \quad (f)$$

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{OH} + p_{N_2} + p_{O_2} + p_{NO} + p_H + p_O = 25; \quad (g)$$

$$\frac{2(p_{H_2O} + p_{H_2}) + p_{OH} + p_H}{p_{CO_2} + p_{CO}} = 3,2; \quad (h)$$

$$\frac{2(p_{CO_2} + p_{O_2}) + p_{CO} + p_{H_2O} + p_{OH} + p_{NO} + p_O}{p_{CO_2} + p_{CO}} = 3,01; \quad (i)$$

$$\frac{2p_{N_2} + p_{NO}}{p_{CO_2} + p_{CO}} = 0,945. \quad (j)$$

The solution of this system of equations is performed for several predetermined temperatures by the method of successive approximation.

Let us set the values of the pressures of those gases present in small quantities in the mixture as equal to zero, namely:

$$p_{OH} = 0, \quad p_{O_2} = 0, \quad p_{NO} = 0, \quad p_H = 0, \quad p_O = 0.$$

Under this assumption, the system of equations takes the form:

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = 25; \quad (g')$$

$$\frac{p_{H_2O} + p_{H_2}}{p_{CO_2} + p_{CO}} = \frac{3.2}{2} = 1.6; \quad (h')$$

$$\frac{2p_{CO_2} + p_{CO} + p_{H_2O}}{p_{CO_2} + p_{CO}} = 3.01; \quad (i')$$

$$\frac{p_{N_2}}{p_{CO_2} + p_{CO}} = \frac{0.945}{2} = 0.4725. \quad (j')$$

The fifth equation is obtained by dividing eq.(a) by eq.(b), whence

$$\frac{p_{CO} \cdot p_{H_2O}}{p_{CO_2} \cdot p_{H_2}} = \frac{K_{p_1}}{K_{p_2}}. \quad (a'b')$$

The solution of this system of equations is performed as follows: From eq.(i'),  $p_{H_2O}$  is expressed in terms of  $p_{CO_2}$  and  $p_{CO}$ :

$$p_{H_2O} = 1.01p_{CO_2} + 2.01p_{CO}.$$

Further, from eq.(h'),  $p_{H_2}$  is expressed in terms of the same quantities by substituting the value of  $p_{H_2O}$ :

$$p_{H_2} = 0.59p_{CO_2} - 0.41p_{CO}.$$

Then, from eq.(j'), we determine  $p_{N_2}$

$$p_{N_2} = 0.4725(p_{CO_2} + p_{CO}).$$

All these values are substituted into eq.(g'), and, in this way, we get the relation between  $p_{CO}$  and  $p_{CO_2}$ :

$$p_{CO} = 8.137 - p_{CO_2}.$$

All partial pressures may now be expressed in terms of  $p_{CO_2}$ :

$$p_{H_2O} = 16.355 - p_{CO_2},$$

$$p_{H_2} = p_{CO_2} - 3.336,$$

$$p_{N_2} = 3.844.$$

The substitution of the expression so obtained into eq.(a'b') leads to a quadratic equation in one unknown: °

$$\frac{(8.137 - p_{CO_2}) \cdot (16.355 - p_{CO_2})}{p_{CO_2} (p_{CO_2} - 3.336)} = \frac{K_{p_1}}{K_{p_2}}.$$

After transformations, this equation is reduced to the form

$$\left(\frac{K_{p1}}{K_{p2}} - 1\right) p_{CO_2}^2 - \left(24,492 - 3,336 \frac{K_{p1}}{K_{p2}}\right) p_{CO_2} - 133,08 = 0.$$

By solving this equation for any temperatures selected, we get only one real root. When this equation is used for various temperatures, only the value of  $\frac{K_{p1}}{K_{p2}}$  varies.

We give below a detailed calculation for the temperature 2800° abs.

From the Table of equilibrium constants (Appendix I) we find for temperature 2800° abs:  $K_{p1} = 0.1565$ ,  $K_{p2} = 0.02233$ ,  $K_{p3} = 0.02091$ ,  $K_{p4} = 0.006649$ ,  $K_{p5} = 0.003345$ ,  $K_{p6} = 0.09374$ ,  $\frac{K_{p1}}{K_{p2}} = 7.008$ .

Consequently, the quadratic equation takes the form:

$$6,008 p_{CO_2}^2 + 1,113 p_{CO_2} - 133,08 = 0.$$

Solution of this equation yields

$$p_{CO_2} = 4,616 \text{ atm abs}$$

since the second root of this equation is  $p_{CO_2} = -5.88$ .

Now we determine the remaining partial pressures:

$$p_{CO} = 8,137 - 4,616 = 3,521 \text{ atm abs}$$

$$p_{H_2O} = 16,355 - 4,616 = 11,739 \text{ atm abs}$$

$$p_{H_2} = 4,616 - 3,336 = 1,280 \text{ atm abs}$$

$$p_{N_2} = 3,844 \text{ atm abs}$$

Thus the composition of the combustion products has been found in first approximation.

Making use of the approximate values of the partial pressures so found, we may determine the composition of the combustion products in second approximation, i.e., we may find:

From eq.(a)

$$p_{O_2} = \left( \frac{K_{p_1} \cdot p_{CO_2}}{p_{CO}} \right)^2 = \left( \frac{0,1565 \cdot 4,616}{3,521} \right)^2 = 0,042 \text{ atm abs}$$

From eq.(c)

$$p_{OH} = \frac{K_{p_1} \cdot p_{H_2O}}{\sqrt{p_{H_2}}} = \frac{0,02091 \cdot 11,739}{\sqrt{1,280}} = 0,217 \text{ atm abs}$$

From eq.(d)

$$p_H = \sqrt{K_{p_1} \cdot p_{H_2}} = \sqrt{0,006649 \cdot 1,280} = 0,092 \text{ atm abs}$$

From eq.(e)

$$p_O = \sqrt{K_{p_2} \cdot p_{O_2}} = \sqrt{0,003345 \cdot 0,042} = 0,012 \text{ atm abs}$$

From eq.(f)

$$p_{NO} = K_{p_3} \sqrt{p_{N_2} \cdot p_{O_2}} = 0,09374 \sqrt{3,844 \cdot 0,042} = 0,038 \text{ atm abs}$$

The quantities so found are substituted into the corresponding original equations, after which these equations are brought into the following form:

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = 24,599, \quad (g'')$$

$$\frac{2p_{H_2O} + 2p_{H_2} + 0,309}{p_{CO_2} + p_{CO}} = 3,2, \quad (h'')$$

$$\frac{2p_{CO_2} + p_{CO} + p_{H_2O} + 0,351}{p_{CO_2} + p_{CO}} = 3,01, \quad (i'')$$

$$\frac{2p_{N_2} + 0,038}{p_{CO_2} + p_{CO}} = 0,945, \quad (j'')$$

From the last three equations, the pressures  $p_{H_2O}$ ,  $p_{H_2}$  and  $p_{N_2}$  are again expressed in terms of the partial pressures  $p_{CO_2}$  and  $p_{CO}$ , as follows:

$$p_{H_2O} = 1,01p_{CO_2} + 2,01p_{CO} - 0,351;$$

$$p_{H_2} = 0,59p_{CO_2} - 0,41p_{CO} + 0,1965;$$

$$p_{N_2} = 0,4725(p_{CO_2} + p_{CO}) - 0,019.$$

These expressions, on substitution into eq.(g'') give, in second approximation, the relation between the partial pressure  $p_{CO}$  and  $p_{CO_2}$ :

$$p_{CO} = 8,063 - p_{CO_2}.$$

Then, by substituting this relation into the preceding equations, we determine

the relation between the remaining partial pressures and the pressure  $p_{CO_2}$ :

$$p_{H_2O} = 15,856 - p_{CO_2},$$

$$p_{H_2} = p_{CO_2} - 3,109,$$

$$p_{N_2} = 3,791.$$

By substituting these relations again in eq. (a'b'), we get:

$$\frac{(8,063 - p_{CO_2}) \cdot (15,856 - p_{CO_2})}{p_{CO_2} (p_{CO_2} - 3,109)} = 7,008$$

or

$$6,008 p_{CO_2}^2 + 2,131 p_{CO_2} - 127,85 = 0.$$

The only satisfactory root of this equation is the partial pressure:

$$p_{CO_2} = 4,439 \text{ atm abs}$$

the remaining pressures, in second approximation, take the following values:

$$p_{CO} = 3,624 \text{ atm abs}$$

$$p_{H_2O} = 11,417 \text{ atm abs}$$

$$p_{H_2} = 1,330 \text{ atm abs}$$

$$p_{N_2} = 3,791 \text{ atm abs}$$

The values of the pressures, in third approximation, are found to be as follows:

$$p_{O_2} = \left( \frac{0,1565 \cdot 4,439}{3,624} \right)^2 = 0,037 \text{ atm abs}$$

$$p_{OH} = \frac{0,02091 \cdot 11,417}{\sqrt{1,330}} = 0,207 \text{ atm abs}$$

$$p_H = \sqrt{0,006649 \cdot 1,330} = 0,094 \text{ atm abs}$$

$$p_O = \sqrt{0,003345 \cdot 0,037} = 0,011 \text{ atm abs}$$

$$p_{NO} = 0,09374 \sqrt{3,791 \cdot 0,037} = 0,035 \text{ atm abs}$$

The equations for determining the remaining partial pressures are somewhat modified, as follows:

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = 24,616, \quad (g' \text{ "})$$

$$\frac{2p_{H_2O} + 2p_{H_2} + 0,301}{p_{CO_2} + p_{CO}} = 3,2, \quad (h' \text{ "})$$



$$\frac{2p_{\text{CO}_2} + p_{\text{CO}} + p_{\text{H}_2\text{O}} + 0,327}{p_{\text{CO}_2} + p_{\text{CO}}} = 3,01, \quad (1' \text{ n})$$

$$\frac{2p_{\text{N}_2} + 0,035}{p_{\text{CO}_2} + p_{\text{CO}}} = 0,945. \quad (j' \text{ n})$$

Consequently, the relations take the following form:

$$p_{\text{H}_2\text{O}} = 1,01p_{\text{CO}_2} + 2,01p_{\text{CO}} - 0,327,$$

$$p_{\text{H}_2} = 0,59p_{\text{CO}_2} - 0,41p_{\text{CO}} + 0,1765,$$

$$p_{\text{N}_2} = 0,4775(p_{\text{CO}_2} + p_{\text{CO}}) - 0,0175.$$

Substituting these equations in eq. (g' n), we get

$$p_{\text{CO}} = 8,066 - p_{\text{CO}_2},$$

whence

$$p_{\text{H}_2\text{O}} = 15,886 - p_{\text{CO}_2},$$

$$p_{\text{H}_2} = p_{\text{CO}_2} - 3,131,$$

$$p_{\text{N}_2} = 3,794.$$

By repeating calculations similar to those in second approximation, we get the equation:

$$6,008p_{\text{CO}_2}^2 + 2,01p_{\text{CO}_2} - 128,14 = 0,$$

whence

$$p_{\text{CO}_2} = 4,454 \text{ atm abs}$$

Consequently,

$$p_{\text{CO}} = 3,612 \text{ atm abs}$$

$$p_{\text{H}_2\text{O}} = 11,432 \text{ atm abs}$$

$$p_{\text{H}_2} = 1,323 \text{ atm abs}$$

$$p_{\text{N}_2} = 3,794 \text{ atm abs}$$

The following approximation gives the values of the partial pressures:

$$p_{\text{O}_2} = \left( \frac{0,1565 \cdot 4,454}{3,612} \right)^2 = 0,037 \text{ atm abs}$$

$$p_{\text{OH}} = \frac{0,02091 \cdot 11,432}{\sqrt{1,323}} = 0,208 \text{ atm abs}$$

$$p_{\text{H}} = \sqrt{0,006649 \cdot 1,323} = 0,094 \text{ atm abs}$$

$$p_{\text{O}} = \sqrt{0,003345 \cdot 0,037} = 0,011 \text{ atm abs}$$

$$p_{\text{NO}} = 0,09374 \sqrt{3,794 \cdot 0,037} = 0,035 \text{ atm abs}$$

The remaining equations now take the form:

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = 24,615;$$

$$\frac{2p_{H_2O} + 2p_{H_2} + 0,302}{p_{CO_2} + p_{CO}} = 3,2,$$

$$\frac{2p_{CO_2} + p_{CO} + p_{H_2O} + 0,328}{p_{CO_2} + p_{CO}} = 3,01,$$

$$\frac{2p_{N_2} + 0,035}{p_{CO_2} + p_{CO}} = 0,945.$$

By repeating the solution of this system we get the quadratic equation

$$6,008p_{CO_2}^2 + 2,016p_{CO_2} - 128,13 = 0,$$

whence

$$p_{CO_2} = 4,453 \text{ atm abs}$$

The pressures of the remaining gases in this case are

$$p_{CO} = 3,613 \text{ atm abs}$$

$$p_{H_2O} = 11,432 \text{ atm abs}$$

$$p_{H_2} = 1,323 \text{ atm abs}$$

$$p_{N_2} = 3,794 \text{ atm abs}$$

By further iteration, we obtain figures that are the same as those already obtained. Consequently our calculation has now been completed.

The composition of the combustion products, in this case, at a temperature of 2800° abs, will be as follows:

$$p_{CO_2} = 4,453, \quad p_{CO} = 3,613, \quad p_{H_2O} = 11,432, \quad p_{H_2} = 1,328, \quad p_{N_2} = 3,794,$$

$$p_{O_2} = 0,037, \quad p_{OH} = 0,208, \quad p_{\bar{H}} = 0,094, \quad p_{O} = 0,011, \quad p_{NO} = 0,035.$$

The following Table is a summary of the results obtained:

	First Approximation	Second Approximation	Third Approximation	Fourth Approximation
$p_{CO_2}$	4,616	4,439	4,454	4,453
$p_{CO}$	3,521	3,624	3,612	3,613
$p_{H_2O}$	11,739	11,417	11,432	11,432
$p_{H_2}$	1,280	1,380	1,323	1,323
$p_{N_2}$	3,844	3,791	3,794	3,794
$p_{O_2}$	0,000	0,042	0,037	0,037
$p_{OH}$	0,000	0,217	0,207	0,208
$p_{\bar{H}}$	0,000	0,092	0,094	0,094
$p_{O}$	0,000	0,012	0,011	0,011
$p_{NO}$	0,000	0,038	0,035	0,035

The molecular weight calculated from eq.(51) for the resultant composition of the combustion products will be

$$\mu_c = \frac{1}{25} [4,453 \cdot 44 + (3,613 + 3,794) 28 + 11,432 \cdot 18 + 1,323 \cdot 2 + 0,037 \cdot 32 + 0,208 \cdot 17 + 0,094 \cdot 1 + 0,011 \cdot 16 + 0,035 \cdot 30] = 24,71.$$

Equation (50) permits determining the energy content of this gas mixture at a temperature of 2800° abs, for which we use the Table in Appendix II.

Computations yield

$$e_s = \frac{10^3}{25 \cdot 24,71} [4,453 \cdot 33,7 + 3,613 \cdot 88,22 + 3,704 \cdot 20,4 + 11,432 \cdot 27,26 + 1,323 \cdot 77,25 + 0,037 \cdot 21,54 + 0,208 \cdot 58,65 + 0,094 \cdot 93,42 + 0,011 \cdot 71,68 + 0,035 \cdot 42,53] = 1593 < 1686.$$

Consequently, the actual temperature of combustion must be higher than assumed.

Let us run a calculation similar to that given above for the temperature of 3000° abs, for which (Appendix II) the equilibrium constants have the following values:

$$K_{p_1} = 0,3417, \quad K_{p_2} = 0,04628, \quad K_{p_3} = 0,04841, \quad K_{p_4} = 0,02475, \\ K_{p_5} = 0,01441, \quad K_{p_6} = 0,1213 \text{ and } \frac{K_{p_1}}{K_{p_2}} = 7,382.$$

In calculating by the usual method of successive approximation at this temperature, only the 11th or 12th approximation give satisfactory results. To accelerate the solution we make use of the suggestion by Instructor V.Ye.Alemasov, Candidate in Technical Sciences, that in first approximation we take the composition of the combustion product obtained for the preceding lower temperature, in this case, for the temperature of 2800° abs; thanks to this procedure, the sixth approximation already yields satisfactory results.

The Table given below, contains a summary of the calculation results.

The molecular weight of the gas mixture so obtained is

$$\mu_s = 24,39,$$

and its energy content is

$$e_s = 1761 \text{ kcal/kg} > 1686.$$

Consequently, the actual temperature of combustion is in the range from 2800 to 3000° abs. To determine this temperature, the interpolation equation (55) is

	Approximation					
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>
$P_{CO_2}$	4,453	4,003	4,149	4,116	4,125	4,122
$P_{CO}$	3,613	3,933	3,818	3,843	3,836	3,839
$P_{H_2O}$	11,432	10,955	11,054	11,021	11,030	11,028
$P_{H_2}$	1,323	1,411	1,377	1,394	1,390	1,390
$P_{N_2}$	3,794	3,700	3,724	3,717	3,719	3,719
$P_{O_2}$	0,037	0,178	0,121	0,137	0,133	0,135
$P_{OH}$	0,208	0,481	0,446	0,456	0,452	0,452
$P_{NO}$	0,035	0,099	0,081	0,087	0,085	0,086
$P_H$	0,094	0,181	0,187	0,184	0,186	0,185
$P_O$	0,011	0,057	0,042	0,044	0,044	0,044

used. To determine the values of M and N we have the following two equations:

$$M + 2800 N = 1593,$$

$$M + 3000 N = 1761;$$

and simultaneous solution of these equations yields the values

$$N = 0,84,$$

$$M = -759.$$

Thus the interpolation equation takes the following form:

$$0,84 T_z - 759 = e,$$

or

$$0,84 T_z = 1686 + 759 = 2445,$$

whence

$$T_z = 2911^\circ \text{ abs}$$

The composition of the combustion products at this temperature is determined by the interpolation method. Assuming a linear temperature dependence of the partial pressures of the component gases, the composition of the combustion products at temperature  $T_z$  is determined from the ratios for each gas.

$$p_i = p_{i1} + (p_{i2} - p_{i1}) \frac{T_z - T_{z1}}{T_{z2} - T_{z1}},$$

where  $T_{z1}$  and  $T_{z2}$  are the temperatures for which the calculation is being made;

$p_{i1}$  and  $p_{i2}$  are the partial pressures of the  $i^{\text{th}}$  gas at these temperatures.

For this case, we have

$$\frac{T_z - T_{z1}}{T_{z2} - T_{z1}} = \frac{2911 - 2800}{3000 - 2800} = 0,555,$$

whence, for carbon dioxide, we get

$$p_{\text{CO}_2} = 4,453 + 0,555 (4,122 - 4,453) = 4,270 \text{ atm abs}$$

and, for carbon monoxide,

$$p_{\text{CO}} = 3,613 + 0,555 (3,839 - 3,618) = 3,738 \text{ atm abs}$$

A calculation so performed gives the following composition of the combustion products for  $T_z = 2911^\circ \text{ abs}$ :

$$\begin{aligned} p_{\text{CO}_2} &= 4,270 \text{ atm abs} & p_{\text{O}_2} &= 0,092 \text{ atm abs} \\ p_{\text{CO}} &= 3,738 \text{ atm abs} & p_{\text{OH}} &= 0,343 \text{ atm abs} \\ p_{\text{H}_2\text{O}} &= 11,208 \text{ atm abs} & p_{\text{H}_2} &= 0,145 \text{ atm abs} \\ p_{\text{H}_2} &= 1,360 \text{ atm abs} & p_{\text{O}} &= 0,029 \text{ atm abs} \\ p_{\text{N}_2} &= 3,752 \text{ atm abs} & p_{\text{NO}} &= 0,063 \text{ atm abs} \end{aligned}$$

The molecular weight of this mixture is found to be

$$\mu_z = 24,53,$$

and the gas constant:

$$R_z = \frac{848}{24,53} = 34,57.$$

Thus the state of this gas mixture is determined by the following parameters:

$$\begin{aligned} \text{pressure} & \quad p_z = 25 \text{ atm abs} \\ \text{temperature} & \quad T_z = 2911^\circ \text{ abs} \\ \text{specific gravity} & \quad \gamma_z = \frac{25 \cdot 1,033 \cdot 10^4}{34,57 \cdot 2911} = 2,566 \text{ kg/m}^3. \end{aligned}$$

$$b. \xi_{\mu_z} = 0,92$$

For purposes of comparison, we shall now calculate the combustion process, allowing for the incomplete liberation of heat as a result of incomplete mixing; for this we take  $\xi_{inc\cdot z} = 0.92$ .

The equation of the energy balance (46) will have the form

$$e_z = e_c - (1 - \xi_{inc}) H_p$$

or

$$e_z = 1686 - (1 - 0.92) 1314 = 1581.$$

The calculation of the composition of the combustion products remains as before. However, since the energy content for a temperature of  $T_z = 2800^\circ$  abs was equal to 1593 kcal/kg, the temperature in this case must be somewhat lower than  $2800^\circ$  abs. In view of the small difference between the energy contents, we shall use the interpolation equation for the calculation. This equation now takes the form

$$-759 + 0.84 T_z = 1581$$

or

$$T_z = 2785^\circ.$$

Thus, owing to the incomplete combustion of the fuel, the combustion temperature is considerably lowered. The composition of the combustion products for this temperature is determined in the same way as in the previous case, by the equation

$$p_i = p_{i1} + \frac{2785 - 2800}{3000 - 2800} (p_{i2} - p_{i1}) = p_{i1} - 0.0075 (p_{i2} - p_{i1}).$$

Consequently,

$$p_{CO_2} = 4.455 \text{ atm abs} \quad p_{O_2} = 0.037 \text{ atm abs}$$

$$p_{CO} = 3.611 \text{ atm abs} \quad p_{OH} = 0.208 \text{ atm abs}$$

$$p_{H_2O} = 11.435 \text{ atm abs} \quad p_H = 0.094 \text{ atm abs}$$

$$p_{H_2} = 1.322 \text{ atm abs} \quad p_O = 0.011 \text{ atm abs}$$

$$p_{N_2} = 3.792 \text{ atm abs} \quad p_{NO} = 0.035 \text{ atm abs}$$

and the molecular weight remains unchanged, as at  $2800^\circ$  abs,

$$\mu_z = 24.71.$$

## 2. Calculation of the Efflux Process

For purposes of comparison, the calculation of the efflux process is calculated under three different assumptions:

1. In the combustion chamber, the combustion process terminates so that  $\xi_{inc z} = 1$ ; the exhaust process is of the extreme nonequilibrium type, i.e., there is no molecular recombination.
2. The exhaust process is in complete equilibrium; the combustion process in the chamber terminates at  $\xi_{inc z} = \xi_{inc a} = 1$ .
3. The combustion process in the chamber is not completed ( $\xi_{inc z} = 0.92$ ), there is partial afterburning of the fuel in the nozzle ( $\xi_{inc a} = 0.98$ ); the process in the nozzle is in complete equilibrium.

### a. $\xi_{inc z} = 1$ ; Exhaust Process of the Extreme Nonequilibrium Type

In this case, the state of the mixture at the exit of the combustion chamber, i.e., before the beginning of efflux, is determined as above; it is determined by the parameters

$$T_s = 2911^\circ \text{ abs}, \quad p_s = 25 \text{ atm abs}, \quad \gamma_s = 2,566 \text{ kg/m}^3.$$

Since the chemical energy of the combustion products in the nozzle remains constant and there is no afterburning, it follows that, in the exhaust process, only conversion of part of the enthalpy of the gases into kinetic energy takes place; a consequence of the adopted condition of energetic equilibrium is that the oscillatory energy of the atoms likewise remains constant; consequently adiabatic flow takes place in the nozzle.

Owing to the fact that the oscillatory energy of the atoms remains constant, the value of the heat capacity and, consequently, that of the adiabatic index, is independent of the temperature and is determined only by the atomicity of the gases and may be calculated on the basis of the molecular-kinetic theory of heat capacity

(Chapter III, Section 5). We find:

$$\begin{aligned}\mu C_v &= \frac{1}{25} [4,96 (3,738 + 1,360 + 3,752 + 0,092 + 0,343 + 0,063) + \\ &\quad + 2,98 (0,145 + 0,029) + 6,71 \cdot 4,270 + 6,80 \cdot 11,208] = 6,07, \\ \mu C_p &= \frac{1}{25} [6,95 (3,738 + 1,360 + 3,752 + 0,092 + 0,343 + 0,063) + \\ &\quad + 4,97 (0,145 + 0,029) + 8,71 \cdot 4,270 + 8,79 \cdot 11,208] = 8,062\end{aligned}$$

or, more simply,

$$\mu C_p = 6,07 + 1,99 = 8,06,$$

whence

$$k = \frac{8,062}{6,07} = 1,328.$$

The temperature at the end of expansion is determined from the fundamental adiabatic equation, i.e.,

$$T_a = T_z \left( \frac{P_a}{P_z} \right)^{\frac{k-1}{k}}$$

or

$$T_a = 2911 \cdot \left( \frac{1}{25} \right)^{\frac{0,328}{1,328}} = 1314^\circ.$$

Since the gas constant is known, the exhaust velocity may be determined; this is equal to

$$w_a = \sqrt{2 \cdot 9,81 \frac{1,328}{0,328} 34,57 (2911 - 1314)} = 2094 \text{ m/sec.}$$

The specific gravity of the gas, on emerging from the nozzle, will be

$$\gamma_a = \frac{1,033 \cdot 10^4}{34,57 \cdot 1314} = 0,228 \text{ kg/m}^3.$$

b. Since  $z = 1$ ; Process of Efflux of the Complete Equilibrium Type

° Owing to the fact that the exhaust process takes place in this case at both chemical and energetic equilibrium, the composition of the combustion product varies, since there is recombination of the molecules in the nozzle corresponding to the temperature decrease. At the same time, the heat capacity decreases, while the



index of polytropy increases. The entire process takes place with a substantial liberation of heat and may be represented as polytropic, with a polytropic index considerably lower than the adiabatic index.

The equation of energy (58) now takes the following form:

$$x_a + i_a + A \frac{w_a^2}{2g} = 1686$$

or

$$e_a + A \frac{w_a^2}{2g} = 1686.$$

To calculate the composition of the combustion product at the end of expansion, the system of equations (65) to (69) is used. This is permissible in this case, since the quantity of monatomic gases in the mixture will be exceedingly small, and the quantity of OH and O<sub>2</sub> will also be small.

The system of equations has the form:

$$\frac{p_{CO} \cdot p_{H_2O}}{p_{CO_2} \cdot p_{H_2}} = \frac{K_{p1}}{K_{p2}}, \quad (a)$$

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{N_2} = 1, \quad (b)$$

$$\frac{p_{H_2O} + p_{H_2}}{p_{CO_2} + p_{CO}} = 1.6, \quad (c)$$

$$\frac{2p_{CO_2} + p_{CO} + p_{H_2O}}{p_{CO_2} + p_{CO}} = 3.01, \quad (d)$$

$$\frac{p_{N_2}}{p_{CO_2} + p_{CO}} = 0.4725. \quad (e)$$

The temperature at the end of expansion must be expected to be considerably higher than in the first case since, as a result of the recombination of molecules, a considerable quantity of heat is liberated.

Let the temperature at the end of expansion be  $T_a = 1800^\circ$  abs.

For this temperature, the Table of Appendix I gives the ratio of the equilibrium constants as  $\frac{K_{p1}}{K_{p2}} = 3.967$ .

The system of equations is solved in the following way:

From eq.(c):

$$p_{H_2O} + p_{H_2} = 1,6 (p_{CO_2} + p_{CO}),$$

From eq.(e):

$$p_{N_2} = 0,4725 (p_{CO_2} + p_{CO}).$$

Substitution of these expressions into eq.(b) gives the relations between  $p_{CO}$  and  $p_{CO_2}$ , which is

$$p_{CO} = 0,3255 - p_{CO_2}.$$

Consequently, the remaining relations will be

From eq.(d):

$$p_{H_2O} = 2,01 (p_{CO_2} + p_{CO}) - p_{CO_2} = 2,01 \cdot 0,3255 - p_{CO_2} = 0,6543 - p_{CO_2};$$

From eq.(c):

$$p_{H_2} = 1,6 (p_{CO_2} + p_{CO}) - p_{H_2O} = 1,6 (p_{CO_2} + p_{CO}) - 2,01 (p_{CO_2} + p_{CO}) + p_{CO_2} = p_{CO_2} - 0,41 (p_{CO_2} + p_{CO}) = p_{CO_2} - 0,41 \cdot 0,3255 = p_{CO_2} - 0,1335;$$

From eq.(e):

$$p_{N_2} = 0,4725 (p_{CO_2} + p_{CO}) = 0,4725 \cdot 0,3255 = 0,1538.$$

After substitution of these relations in eq.(a), the latter takes the form:

$$\frac{(0,3255 - p_{CO_2})(0,6543 - p_{CO_2})}{p_{CO_2}(p_{CO_2} - 0,1335)} = 3,967$$

or

$$2,965 p_{CO_2}^2 + 0,4502 p_{CO_2} - 0,213 = 0,$$

whence

$$p_{CO_2} = 0,2026.$$

Consequently, for  $T_a = 1800^\circ$  abs, the composition of the combustion product will be as follows:

$$p_{CO_2} = 0,2026 \text{ atm abs}$$

$$p_{CO} = 0,1229 \text{ atm abs}$$

$$p_{H_2O} = 0,4517 \text{ atm abs}$$

$$p_{H_2} = 0,0690 \text{ atm abs}$$

$$p_{N_2} = 0,1538 \text{ atm abs}$$

$$p_a = 1,000 \text{ atm abs}$$

The molecular weight of this mixture will be

$$u_a = 0,2026 \cdot 44 + (0,1229 + 0,1538) 28 + 0,4517 \cdot 18 + 0,069 \cdot 2 = 24,93$$

and the gas constants will be

$$R_a = \frac{848}{24,93} = 34,02.$$

From Appendix II we determine the energy content of the mixture for the temperature of the calculations:

$$e_a = \frac{10^6}{24,93} [0,2026 \cdot 19,01 + 0,1229 \cdot 79,47 + 0,4517 \cdot 14,93 + 0,069 \cdot 68,83 + 0,1538 \cdot 11,71] = 1079 \text{ kcal / kg.}$$

To calculate the kinetic energy of the gases we determine the index of polytropy from eq.(64):

$$n = \frac{\lg 25}{\lg 25 \frac{1800}{2911}} = 1,1804.$$

The kinetic energy in the mixture in this case is equal to

$$A \frac{w_a^2}{2g} = A \frac{n}{n-1} \frac{R_z + R_a}{2} T_z \left[ 1 - \left( \frac{1}{25} \right)^{\frac{n-1}{n}} \right] =$$

$$= \frac{1,1804 (34,57 + 34,02) 2911}{427 \cdot 0,1804 \cdot 2} \left[ 1 - \left( \frac{1}{25} \right)^{\frac{0,1804}{1,1804}} \right] = 594 \text{ kcal / kg.}$$

Thus, for the temperature  $T_a = 1800^\circ$  abs, the total energy content (chemical energy + calorific energy + kinetic energy) will be

$$e_a + A \frac{w_a^2}{2g} = 1079 + 594 = 1673 < 1686.$$

Thus, the actual temperature at the end of expansion must be somewhat higher than  $1800^\circ$  abs.

Calculations made for a temperature of  $1900^\circ$  abs give the following composition of the combustion products:

$$\begin{aligned} p_{CO_2} &= 0,1994 \text{ atm abs} \\ p_{CO} &= 0,1261 \text{ atm abs} \\ p_{H_2O} &= 0,4548 \text{ atm abs} \\ p_{H_2} &= 0,0659 \text{ atm abs} \\ p_{N_2} &= 0,1538 \text{ atm abs} \\ \hline p_a &= 1,0000 \text{ atm abs} \end{aligned}$$

The molecular weight remains unchanged at

$$\mu_c = 24,93.$$

The energy content calculated from Appendix II is

$$e_a = 1125 \text{ kcal / kg}$$

The index of polytropy is

$$n = 1,1574$$

and the kinetic energy is

$$A \frac{w_a^2}{2g} = 617 \text{ kcal / kg}$$

Thus the total energy content of the mixture at  $T_a = 1900^\circ$  abs is

$$e_a + A \frac{w_a^2}{2g} = 1125 + 617 = 1742 \text{ kcal / kg}$$

Interpolation gives the actual temperature at the end of the nozzle:

$$T_a = 1820^\circ.$$

The composition of the combustion product is found to be as follows:

$$p_{CO_2} = 0,2020 \text{ atm abs}$$

$$p_{CO} = 0,1235 \text{ atm abs}$$

$$p_{H_2O} = 0,4511 \text{ atm abs}$$

$$p_{H_2} = 0,0696 \text{ atm abs}$$

$$p_{N_2} = 0,1538 \text{ atm abs}$$

The gas constant will then be

$$R_a = 34,02.$$

The index of polytropy is

$$n = 1,171.$$

Consequently, the exhaust velocity is equal to

$$w_a = \sqrt{2 \cdot 9,81 \cdot \frac{1,171}{0,171} \cdot \frac{34,02 + 34,57}{2} \cdot (2911 - 1820)} = 2260 \text{ m/sec}$$

The specific weight of the mixture, at emergence from the nozzle, is

$$\gamma_a = \frac{1,033 \cdot 10^4}{34,02 \cdot 1820} = 0,167 \text{ kg/m}^3.$$

c.  $\xi_{inc z} = 0.92, \xi_{inc a} = 0.98$ ; Process in Complete Equilibrium

In this case the initial state of the combustion products is defined by the temperature  $2785^\circ$  abs and the gas constant is

$$R_z = \frac{848}{24.71} = 34.32.$$

The equation of energy balance (58) has the form:

$$e_a + A \frac{w_a^2}{2g} = 1686 - (1 - 0.98) 1314 = 1660.$$

The calculation of the composition of the combustion product is the same as the calculation given in the preceding case.

For a temperature of  $1800^\circ$  abs, the energy content was  $e_a = 1079$  kcal/kg. The kinetic energy will have a different value, since the index of polytropy must be recalculated:

$$n = \frac{\lg 25}{\lg 25 \frac{1800}{2785}} = 1.157$$

and the kinetic energy in this case is known to be

$$A \frac{w_a^2}{2g} = \frac{1.157 (34.32 + 34.57) 2785}{0.157 \cdot 427 \cdot 2} \left[ 1 - (0.04)^{\frac{0.157}{1.157}} \right] = 568 \text{ kcal / kg}$$

The total energy content of the mixture is

$$e_a + A \frac{w_a^2}{2g} = 1079 + 586 = 1665 \text{ kcal / kg}$$

A calculation made for  $T_a = 1900^\circ$  abs gives the following results:

$$n = 1.135,$$

$$A \frac{w_a^2}{2g} = 680 \text{ kcal / kg}$$

and a total energy content:

$$e_a + A \frac{w_a^2}{2g} = 1125 + 680 = 1805 \text{ kcal / kg}$$

By means of the interpolation equation we find that the actual temperature at the end of the discharge will be:

$$T_e = 1796^\circ \text{ abs}$$

Since the temperature at the end of efflux differs very slightly from  $1800^\circ \text{ abs}$ , it may be taken, in first approximation, that the composition of the combustion products will in this case be

$$\begin{aligned} p_{\text{CO}_2} &= 0,1994 \text{ atm abs} \\ p_{\text{CO}} &= 0,1261 \text{ atm abs} \\ p_{\text{H}_2\text{O}} &= 0,4548 \text{ atm abs} \\ p_{\text{H}_2} &= 0,0659 \text{ atm abs} \\ p_{\text{N}_2} &= 0,1538 \text{ atm abs} \end{aligned}$$

The index of polytropy is slightly changed, and is

$$n = 1,1577$$

while the exhaust velocity will be

$$w_e = 2229 \text{ m/sec}$$

For purposes of comparison, the Table gives the results of the calculation under all three assumptions.

	Nonequilibrium $\xi_{\text{noz}} = 1$	Equilibrium $\xi_{\text{noz}} = 1$	Equilibrium $\xi_{\text{noz}} = 0,92, \xi_{\text{noz}} = 0,98$
$T_z^\circ \text{ abs}$	2911	2911	2785
$\gamma_z \text{ kg/m}^3$	2,566	2,566	2,701
$\mu_z$	24,53	24,53	24,71
$R_z$	34,57	34,57	34,32
$T_a^\circ \text{ abs}$	1314	1820	1796
$\gamma_a \text{ kg/m}^3$	0,228	0,167	0,169
$\mu_a$	24,53	24,93	24,94
$R_a$	34,57	34,02	34,01
$w_a \text{ m/sec}$	2094	2260	2229

Example 6. Calculate the processes in the combustion chamber and the nozzle of a liquid-propellant rocket engine for the following propellant: ethyl alcohol of

concentration 93.5% ( $C_2H_6O \cdot 0.178 H_2O$ ) and liquid oxygen, at a coefficient of excess oxidizer of  $\alpha = 0.82$ ; a pressure in the combustion chamber  $p_z = 35$  atm abs, and a pressure on the cross section of the nozzle  $p_a = 1$  atm abs.

a. Calculation of the Combustion Processes

The calculation is run at  $\xi_{inc z} = \xi_{inc a} = 1$ .

For the fuel components we have the following characterizing data:

Ethyl alcohol, 93.5%:

chemical formula  $C_2H_6O \cdot 0.178 H_2O$ ,

specific gravity at 25° abs,  $\gamma_2 = 0.8028$  kg/ltr,

heat value  $H_{\mu c} = 294,300$  kcal/mole,

weight of 1 mole  $\mu_c^! = 49.2$  kg/mole.

Liquid oxygen:

specific gravity  $\gamma_o = 1.14$  kg/ltr,

heat value  $H_{\mu o} = 0$ ,

heat of vaporization and heating to 25° abs, 3030 kcal/mole

We obtain the following characteristics for the fuel under the specified conditions.

Stoichiometric coefficients of the component ratio:

$$k_o = \frac{2 \cdot 2 + 0.5 \cdot 6 - 1}{2} = 3 \text{ mole / mole}$$

Molecular ratio of the components at  $\alpha = 0.82$ :

$$ak_o = 0.82 \cdot 3 = 2.46 \text{ mole / mole}$$

Weight ratio:

$$\alpha k_o' = 2.46 \cdot \frac{32}{46} \cdot \frac{93.5}{100} = 1.60 \text{ kg/kg.}$$

Volume ratio:

$$\alpha k_o'' = 1.6 \cdot \frac{0.8028}{1.14} = 1.127 \text{ ltr / ltr}$$

Composition of fuel by weight:

$$g_c = \frac{1}{1 + 1,6} = 0,385,$$

$$g_o = \frac{1,6}{1 + 1,6} = 0,615.$$

Composition of fuel by volume:

$$r_c = \frac{1}{1 + 1,127} = 0,47,$$

$$r_o = \frac{1,127}{1 + 1,127} = 0,53.$$

Specific gravity of fuel:

$$\gamma = \frac{1}{\frac{0,385}{0,8028} + \frac{0,615}{1,14}} = 0,981 \text{ kg/ltr}$$

Molecular heat content of fuel:

$$H_u = 0,82(294\,300 - 3 \cdot 3030) = 233\,870 \text{ kcal / mole}$$

Heat value by weight:

$$H_u = \frac{233\,870}{49,2 + 2,46 \cdot 32} = 1830 \text{ kcal / kg}$$

Heat value by volume:

$$H_v = 1830 \cdot 0,981 = 1795 \text{ kcal / ltr}$$

Molecular chemical energy:

$$X = 294\,300 - 3 \cdot 0,82 \cdot 3030 = 286\,850 \text{ kcal / mole}$$

Chemical energy by weight:

$$x_s = \frac{286\,850}{49,2 + 2,46 \cdot 32} = 2\,240 \text{ kcal / kg}$$

The equation of energy for the exit of the combustion chamber takes the form:

$$e_s = e_c = x_c + i_c = 2240 + i_c.$$

In this case, the initial enthalpy is

$$i_c = \frac{0,71 \cdot 25 - 1,6 \frac{3030}{32}}{2,6} = -52 \text{ kcal / kg}$$



The computational equation for the combustion process will have the form:

$$e_s = x_s + L_s = 2240 - 52 = 2188 \text{ kcal/kg}$$

The system of computational equations consists of eight such equations, owing to the absence of nitrogen from the fuel. They are as follows:

$$\frac{p_{\text{CO}} \cdot p_{\text{O}_2}^{0.5}}{p_{\text{CO}_2}} = K_{p,1}, \quad (\text{a})$$

$$\frac{p_{\text{H}_2} \cdot p_{\text{O}_2}^{0.5}}{p_{\text{H}_2\text{O}}} = K_{p,2}, \quad (\text{b})$$

$$\frac{p_{\text{OH}} \cdot p_{\text{H}_2}^{0.5}}{p_{\text{H}_2\text{O}}} = K_{p,3}, \quad (\text{c})$$

$$\frac{p_{\text{H}}^2}{p_{\text{H}_2}} = K_{p,4}, \quad (\text{d})$$

$$\frac{p_{\text{O}}^2}{p_{\text{O}_2}} = K_{p,5}, \quad (\text{e})$$

$$p_{\text{CO}_2} + p_{\text{CO}} + p_{\text{H}_2\text{O}} + p_{\text{H}_2} + p_{\text{OH}} + p_{\text{O}} + p_{\text{H}} + p_{\text{O}_2} = 35, \quad (\text{f})$$

$$\frac{2(p_{\text{H}_2\text{O}} + p_{\text{H}_2}) + p_{\text{OH}} + p_{\text{H}}}{p_{\text{CO}_2} + p_{\text{CO}}} = \frac{6 + 2 \cdot 0,178}{2} = 3,178, \quad (\text{g})$$

$$\frac{2(p_{\text{CO}_2} + p_{\text{O}_2}) + p_{\text{CO}} + p_{\text{H}_2\text{O}} + p_{\text{OH}} + p_{\text{O}}}{p_{\text{CO}_2} + p_{\text{CO}}} = \frac{1 + 0,178 + 2,46 \cdot 2}{2} = 3,229. \quad (\text{h})$$

In the propellant which is being calculated in this example, oxygen is the oxidizer, so that its combustion temperature must be very high, above  $3000^\circ \text{ abs.}$  The calculation of the combustion products at such high temperatures by the method of successive approximation requires an excessive amount of time; owing to the slow approximation of the results, the calculation of the system of equations must be repeated 20 - 25 times or more; the method of preliminary selection of the partial pressure of oxygen leads much more rapidly to the results, especially if we use the method proposed by Instructor V.Ye.Alemašov to solve the system of equations.

Let it be assumed that the oxygen pressure in the mixture is taken as  $p_{\text{O}_2} = a^2$ .

Then all the remaining pressures in the equations of chemical equilibrium may be expressed in terms of the pressures  $p_{\text{CO}_2}$  and  $p_{\text{H}_2\text{O}}$ , as follows:

$$\begin{aligned}
\text{From eq. (a),} \quad p_{\text{CO}} &= \frac{K_{p_1}}{a} p_{\text{CO}_2}, \\
\text{From eq. (b),} \quad p_{\text{H}_2} &= \frac{K_{p_2}}{a} p_{\text{H}_2\text{O}}, \\
\text{From eq. (c),} \quad p_{\text{OH}} &= K_{p_3} \sqrt{\frac{a}{K_{p_1}}} \cdot \sqrt{p_{\text{H}_2\text{O}}}, \\
\text{From eq. (d),} \quad p_{\text{H}} &= \sqrt{K_{p_4}} \cdot \sqrt{\frac{K_{p_1}}{a}} \cdot \sqrt{p_{\text{H}_2\text{O}}}, \\
\text{From eq. (e),} \quad p_{\text{O}} &= \sqrt{K_{p_5}} \cdot a.
\end{aligned}$$

Introducing the following notation, for simplicity:

$$\begin{aligned}
\frac{K_{p_1}}{a} &= C, \\
\frac{K_{p_2}}{a} &= e, \\
K_{p_3} \sqrt{\frac{a}{K_{p_1}}} &= \frac{K_{p_1}}{\sqrt{e}} = h, \\
K_{p_4} \sqrt{\frac{K_{p_1}}{a}} &= K_{p_4} \sqrt{e} = m, \\
a \sqrt{K_{p_5}} &= b,
\end{aligned}$$

the following relations may be written:

$$\begin{aligned}
p_{\text{CO}_2} + p_{\text{CO}} &= (1 + C) p_{\text{CO}_2}, \\
p_{\text{H}_2\text{O}} + p_{\text{H}_2} &= (1 + e) p_{\text{H}_2\text{O}}, \\
p_{\text{OH}} + p_{\text{H}} &= (h + m) \sqrt{p_{\text{H}_2\text{O}}}.
\end{aligned}$$

Thus eq. (g) takes the form:

$$2(1 + e) p_{\text{H}_2\text{O}} + (h + m) \sqrt{p_{\text{H}_2\text{O}}} = A(1 + C) p_{\text{CO}_2}.$$

Equation (h), after transformations, will have the form:

$$p_{\text{H}_2\text{O}} + h \sqrt{p_{\text{H}_2\text{O}}} + 2p_{\text{O}_2} + p_{\text{O}} = [(B - 1)(1 + C) - 1] p_{\text{CO}_2}.$$

On dividing eq. (g) by eq. (h), we get

$$\frac{2(1 + e) p_{\text{H}_2\text{O}} + (h + m) \sqrt{p_{\text{H}_2\text{O}}}}{p_{\text{H}_2\text{O}} + h \sqrt{p_{\text{H}_2\text{O}}} + 2p_{\text{O}_2} + p_{\text{O}}} = \frac{A(1 + C)}{(B - 1)(1 + C) - 1} = d,$$

where

$$d = \frac{A(1 + C)}{(B - 1)(1 + C) - 1}.$$

After transformations, this equation takes the form:

$$f(V\sqrt{p_{H,O}})^2 - n\sqrt{p_{H,O}} - dr = 0,$$

where

$$f = 2(1+e) - d,$$

$$n = h(d-1) - m,$$

$$r = 2a^2 + b.$$

The equation has only a single positive root:

$$\sqrt{p_{H,O}} = \frac{n + \sqrt{n^2 + 4fdr}}{2f}.$$

After finding the value of  $\sqrt{p_{H,O}}$ , all the remaining quantities are determined by the relations given above; the sum of all the pressures, according to eq.(f), must equal the pressure in the combustion chamber  $p_z$ . In selecting the partial pressure of oxygen, a rough guide can be the fact that it is proportional to the sum found for the pressures,  $p_z$ . The calculation is more conveniently performed by tabulating the results of the computations.

We give below the calculation for a temperature of 3300° abs; for this temperature, we find from Appendix I the equilibrium constants given in the Table below.

$$K_{p1} = 0.9179, K_{p2} = 0.1173, K_{p3} = 0.141, K_{p4} = 0.1324, K_{p5} = 0.09253.$$

As shown in the Table, at  $p_{O_2} = 0.95$  atm abs, the pressure of the mixture  $\Sigma p_i$  is 26.694 atm abs; at  $p_{O_2} = 1.21$  atm abs,  $\Sigma p_i = 40.094$  atm abs. Thus,  $p_{O_2}$  is between 0.95 atm abs and 1.21 atm abs. The calculation rapidly yields the result.

Thus the composition of the combustion products for a temperature of 3300° abs was found to be as follows:

$$\begin{aligned} p_{CO_2} &= 6,707 \text{ atm abs} \\ p_{CO} &= 5,821 \text{ atm abs} \\ p_{H_2O} &= 16,916 \text{ atm abs} \\ p_{H_2} &= 1,871 \text{ atm abs} \\ p_{OH} &= 1,744 \text{ atm abs} \\ p_{O_2} &= 1,121 \text{ atm abs} \\ p_{H} &= 0,498 \text{ atm abs} \\ p_{O} &= 0,322 \text{ atm abs} \\ \hline \Sigma p_i &= 35,000 \text{ atm abs} \end{aligned}$$

Table of Computations for 3300° abs

Equilibrium Constants:  $K_{p1} = 0.9179$ ,  $K_{p2} = 0.1173$ ,  $K_{p3} = 0.1410$ ,  
 $K_{p4} = 0.1324$ ,  $K_{p5} = 0.09259$

$P_{O_2} = a^2$	0,95	1,21	1,11	1,122
$a$	0,975	1,100	1,054	1,059
$p_{O_2} = b = a \sqrt{K_{p_1}}$	0,296	0,334	0,320	0,322
$\frac{K_{p_1}}{a} = C$	0,942	0,835	0,871	0,868
$1+C$	1,942	1,835	1,871	1,868
$A(1+C)$	6,172	5,832	5,946	5,937
$(B-1)(1+C)-1$	3,329	3,090	3,170	3,164
$\frac{A(1+C)}{(B-1)(1+C)-1} = d$	1,854	1,887	1,876	1,876
$\frac{K_{p_2}}{a} = e$	0,120	0,106	0,111	0,1106
$1+e$	1,120	1,106	1,111	1,1196
$2(1+e)$	2,240	2,212	2,222	2,2212
$2(1+e)-d=f$	0,386	0,325	0,346	0,3452
$\sqrt{e}$	0,346	0,326	0,332	0,3326
$\frac{K_{p_3}}{\sqrt{e}} = h$	0,403	0,433	0,425	0,424
$h(d-1)$	0,348	0,384	0,372	0,371
$\sqrt{K_{p_4}} \sqrt{e} = m$	0,126	0,118	0,121	0,121
$h(d-1)-m=n$	0,222	0,266	0,251	0,250
$n^2$	0,049	0,071	0,063	0,0625
$b+2a^2=r$	2,196	2,754	2,540	2,562
$4fdr$	6,286	6,756	6,594	6,636
$\sqrt{n^2+4fdr}$	2,517	2,613	2,580	2,588
$n+\sqrt{n^2+4fdr}=S$	2,739	2,879	2,831	2,838
$\frac{S}{2f} = \sqrt{P_{H_2O}}$	3,548	4,429	4,091	4,113
$P_{H_2O}$	12,586	19,612	16,738	16,914
$2P_{H_2O}$	25,172	39,224	33,476	33,828
$P_{H_2} = eP_{H_2O}$	1,510	2,079	1,858	1,871
$2P_{H_2}$	3,020	4,158	3,716	3,742
$P_{OH} = h \sqrt{P_{H_2O}}$	1,448	1,918	1,739	1,744

(Cont'd)

$p_{O_2}=a^2$	0,95	1,21	1,11	1,122
$p_H=m \sqrt{p_{H_2O}}$	0,447	0,523	0,495	0,498
$2p_{H_2O}+2p_{H_2}+p_{OH}+p_H=t$	30,087	45,823	39,426	39,812
$\frac{t}{A(1+C)}=p_{CO_2}$	4,875	7,857	6,631	6,706
$p_{CO}=Cp_{CO_2}$	4,582	6,561	5,776	5,821
$\sum p_i$	26,694	40,094	34,667	34,997

The molecular weight of the combustion products of such a mixture will be

$$\mu_x=23,93.$$

Their energy content is equal to (Appendix II)

$$e_x=l_x+x_x=2090 \text{ kcal/kg}$$

Since the energy content of the gases is found to be smaller than the energy content of the fuel, a similar calculation is run for a higher temperature, namely for 3400° abs. The result of this calculation yields the following composition of the combustion products:

$$p_{CO_2}=5,998 \text{ atm abs}$$

$$p_{CO}=6,271 \text{ atm abs}$$

$$p_{H_2O}=15,972 \text{ atm abs}$$

$$p_{H_2}=2,108 \text{ atm abs}$$

$$p_{OH}=2,132 \text{ atm abs}$$

$$p_{O_2}=1,375 \text{ atm abs}$$

$$p_H=0,676 \text{ atm abs}$$

$$p_O=0,468 \text{ atm abs}$$

$$\sum p_i=35,000 \text{ atm abs}$$

The molecular weight for this case is found to be

$$\mu_x=23,42,$$

and the energy content,

$$e_s = 2259 \text{ kcal./kg}$$

Consequently, the actual combustion temperature is in the range of 3300 to 3400° abs; it is determined from the interpolation equation and is equal to

$$T_s = 3358^\circ \text{ abs}$$

The composition of the combustion products at this temperature is determined by interpolation and is found to be as follows:

$$\begin{array}{r} p_{\text{CO}_2} = 6,293 \text{ atm abs} \\ p_{\text{CO}} = 6,081 \text{ atm abs} \\ p_{\text{H}_2\text{O}} = 16,358 \text{ atm abs} \\ p_{\text{H}_2} = 2,023 \text{ atm abs} \\ p_{\text{OH}} = 1,969 \text{ atm abs} \\ p_{\text{O}_2} = 1,268 \text{ atm abs} \\ p_{\text{H}} = 0,601 \text{ atm abs} \\ p_{\text{O}} = 0,407 \text{ atm abs} \\ \hline \sum p_i = 35,000 \text{ atm abs} \end{array}$$

The actual molecular weight is

$$\mu_s = 23,63 \text{ and } e_s = 2188 \text{ kcal./kg.}$$

#### b. Calculation of the Process of Efflux

The computational equation of the energy balance has the form:

$$i_a + x_a + A \frac{w_a^2}{2g} = 2188$$

or

$$e_a + A \frac{w_a^2}{2g} = 2188.$$

Owing to the lower exhaust temperatures, the calculation may be performed by the method of successive approximation, which in this case rapidly leads to the result.

The system of equations is reduced to the form:

$$\frac{p_{CO} \cdot p_{O_2}^{0.5}}{p_{CO_2}} = K_{p_1},$$

$$\frac{p_{H_2} \cdot p_{O_2}^{0.5}}{p_{H_2O}} = K_{p_2},$$

$$\frac{p_{OH} \cdot p_{H_2}^{0.5}}{p_{H_2O}} = K_{p_3},$$

$$\frac{p_H^2}{p_{H_2}} = K_{p_4},$$

$$\frac{p_O^2}{p_{O_2}} = K_{p_5},$$

$$p_{CO_2} + p_{CO} + p_{H_2O} + p_{H_2} + p_{OH} + p_{O_2} + p_H + p_O = 1,$$

$$\frac{2(p_{H_2O} + p_{H_2}) + p_{OH} + p_H}{p_{CO_2} + p_{CO}} = 3,178,$$

$$\frac{2(p_{CO_2} + p_{O_2}) + p_{CO} + p_{H_2O} + p_{OH} + p_O}{p_{CO_2} + p_{CO}} = 3,229.$$

A calculation similar to that given in the preceding example rapidly leads to the results.

The temperature of the exhaust gases is found to be  $T_a = 2464^\circ$  abs, and the composition of the combustion products in this case is

$$p_{CO_2} = 0,265 \text{ atm abs} \quad p_{OH} = 0,014 \text{ atm abs}$$

$$p_{CO} = 0,115 \text{ atm abs} \quad p_{O_2} = 0,006 \text{ atm abs}$$

$$p_{H_2O} = 0,556 \text{ atm abs} \quad p_H = 0,004 \text{ atm abs}$$

$$p_{H_2} = 0,039 \text{ atm abs} \quad p_O = 0,001 \text{ atm abs}$$

Their molecular weight is found to be

$$\mu = 25,39.$$

Thus the gas constant at the beginning of efflux was  $R_z = 34.64$  and, on emergence from the nozzle,

$$R_s = \frac{848}{25,39} = 33,4.$$

To determine the exhaust velocity, the following mean value may be taken for

the gas constant:

$$R = \frac{34,64 + 33,4}{2} = 34,02.$$

The value of the index of polytropy under these conditions is found to be

$$n = \frac{\lg 35}{\lg 35 \frac{2464}{3358}} = 1,095.$$

Consequently, the exhaust velocity will be

$$w_a = \sqrt{2 \cdot 9,81 \frac{1,095}{0,095} 34,02 (3358 - 2464)} = 2623 \text{ m/sec.}$$

The parameters characterizing the combustion products are as follows:

	Before Efflux	After Efflux
Pressure	35 atm abs	1 atm abs
Temperature	3358° abs	2464° abs
Specific gravity	3.108 kg/m <sup>3</sup>	0.1265 kg/m <sup>3</sup>



CHAPTER V  
TECHNIQUE OF CONSTRUCTING DIAGRAMS FOR CALCULATING THE PROCESSES  
OF COMBUSTION AND DISCHARGE OF THE COMBUSTION PRODUCTS

1. General Methods of Constructing the Diagrams

Regardless of the methods used for calculating the processes of combustion and discharge of the combustion products at the high combustion temperatures developed in liquid-propellant rocket engines as a result of the use of liquid oxidizers, the greatest difficulties arise in determining the composition of the combustion products. A large amount of time and labor is spent on this calculation. Yet it is necessary to know the composition of the combustion products in order to calculate their molecular weight, enthalpy, and unutilized chemical energy at specified temperatures. The question arises whether it is possible to conduct the calculation of the combustion and discharge processes from diagrams constructed in advance.

A considerable number of enthalpy diagrams have been proposed, permitting the calculation without determining the composition of the combustion products. But the problem is solved on these diagrams only for adiabatic discharge. To obtain results closer to reality, a number of coefficients entirely without physical meaning have been introduced.

Such coefficients, which bring the results of calculation closer to reality, give no clear picture of the processes that take place and offer no information on the physical nature of the phenomena; therefore, they must be regarded as a temporary auxiliary aid for the calculation. It is necessary to take account of the fact that

the discharge processes differ greatly from adiabatic flow. This is due to the partial afterburning of the fuel and the partial recombination of the molecules taking place during passage of the gases through the nozzle. In reality, the discharge process is a process involving the supply of considerable additional quantities of heat, i.e., a polytropic process with an index of polytropy nearer to unity than to the adiabatic index  $k$ .

The calculation method given in Chapter III makes it possible to construct computation diagrams based on principles totally different from the usual IS-diagrams. We shall now give a description of the method of constructing such diagrams, based on the above method of calculation.

## 2. Computational Diagrams for $\xi_{inc} z < 1$ and $\xi_{inc} a < 1$

The calculation begins with the determination of the chemical energy introduced by the propellant into the combustion chamber, and the heat value of this propellant. To determine the chemical energy of the propellant we use eq.(33):

$$x_c = \frac{H_{\mu_c} + \alpha k_o H_{\mu_o}}{\mu_c + \alpha k_o \mu_o} \text{ Kcal / Kg,}$$

where  $x_c$  is the chemical energy of 1 kg of fuel at the specified concentrations of the components and at the assumed coefficient of excess oxidizer  $\alpha$ . To determine the heat value of the propellant, eq.(31) is used:

$$H_p = \frac{(H_{\mu_c} + k_o H_{\mu_o}) \alpha}{\mu_c + \alpha k_o \mu_o} \text{ Kcal / Kg.}$$

The quantity of heat introduced into the combustion chamber by 1 kg of fuel, i.e., the initial enthalpy of the fuel, will be

$$i_c = \frac{(\mu_c' c_c' + \alpha k_o \mu_o' c_o') t_c}{\mu_c' + \alpha k_o \mu_o'} \text{ Kcal / Kg}$$

where  $t_c$  is the temperature of the liquid components on their introduction into the

combustion chamber,  $c_c^i$  and  $c_o^i$  are the heat capacities of the liquid components at the specified concentration.

Denoting the enthalpy of the combustion products at the combustion temperature, and the chemical energy remaining unutilized at the exit of the combustion chamber, by  $i_z$  and  $x_z$  respectively, eq.(46) may be written as follows:

$$i_z + x_z = x_c + i_c - (1 - \xi_{incz}) H_p$$

or

$$e_z = e_c - (1 - \xi_{incz}) H_p.$$

In this equation,  $i_c$ ,  $x_c$ , and  $H_p$  are known, while the coefficient of heat liberation in the combustion chamber,  $\xi_{incz}$  must be taken on the basis of experimental data.

The right side of the equation may be calculated and represented on a diagram in which the values of  $\xi_{incz}$  are plotted on the abscissa and the values of  $x_c + i_c - (1 - \xi_{incz}) H_p$ , calculated for various values of  $\alpha$  and various concentrations of the components, are plotted on the ordinate. Rough calculations show that this part of the diagram (I) will consist of a family of inclined straight lines whose slope is flatter the lower the concentration of the components, i.e., the more water there is in the fuel. Thus this part of the diagram will be roughly of the shape shown in Fig.3. In this diagram, the concentrations  $\sigma_1$  are greater than the concentrations  $\sigma_2$  and the values of  $\alpha_2$  are greater than the values of  $\alpha_1$ .

Thus this part of the diagram completely covers all combinations for one and the same fuel at any concentrations of the components and for any values of the coefficient of excess oxidizer  $\alpha$ .

The construction of the part II of the diagram, i.e., of the value of  $e_z = i_z + x_z$ , is the most time-consuming and difficult, since for constructing these curves the composition of the combustion products at the exit of the combustion chamber must be determined at various assumed combustion temperatures; also, the excessive dissociation must be taken into account in the calculation.

In the general case, only the gases present in the gas mixture in more or less high concentrations are usually taken into account in the combustion products, i.e.,  $\text{CO}_2$ ,  $\text{CO}$ ,  $\text{H}_2\text{O}$ ,  $\text{H}_2$ ,  $\text{N}_2$ ,  $\text{O}_2$ ,  $\text{NO}$ ,  $\text{OH}$ ,  $\text{O}$  and  $\text{H}$ , the composition of the combustion products being determined in terms of partial pressures for convenience of calculation.

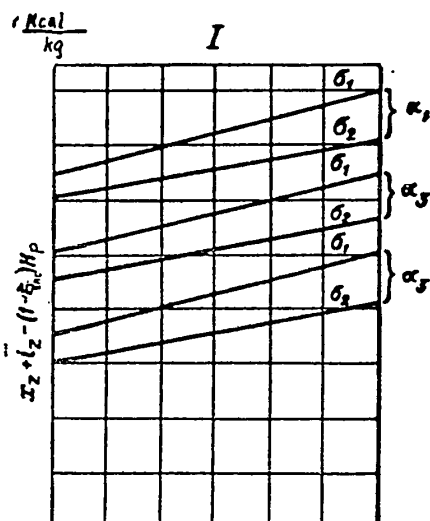


Fig.3 - Diagram of  $x_2 + i_2 - (1 - \xi_{\text{inc } z}) H_p$  to  $\xi_{\text{inc } z}$  as a Function of  $\alpha$  and  $\sigma$

The calculation system of equations is given in Chapter III, which also contains a detailed exposition of the solution of this system.

After determining the composition of the combustion products for a definite temperature at a definite excess oxidizer coefficient, we calculate their molecular weight on the basis of eq.(50), and then, from eqs.(48) and (49), we determine their enthalpy and the chemical energy remaining unutilized. The values obtained for this sum are plotted on the diagram II, where the values of the combustion temperature are plotted on the abscissa, while

the ordinate coincides with the ordinate of the diagram I, i.e., the kcal/kg values are plotted on it. Rough calculations have shown that even at considerable variations in the concentrations of the components, the values of  $i_2 + x_2$  for definite values of  $\alpha$  and for a specified temperature differ so slightly (in a range of 2 - 3 kcal/kg) that this variation may be freely neglected. The values of the molecular weight of the combustion product are also plotted on this same part of the diagram. This second part of the diagram is shown in Fig.4 in combination with diagram I. The curves for the values of  $i_2 + x_2$  rise considerably with increasing temperature, and their level is lower the greater the coefficient of excess oxidizer, since with an increasing value of  $\alpha$  the combustion becomes more complete and since,

for this reason, at a constant value of  $i_z + x_z$ , the temperature continually rises. The curves characterizing the molecular weight of the combustion products decline with increasing temperature, since at higher temperatures the gases are more strongly dissociated, causing the appearance of lighter diatomic and even monatomic gases in the gas mixture.

When these two parts of the diagram are combined, it becomes possible to cal-

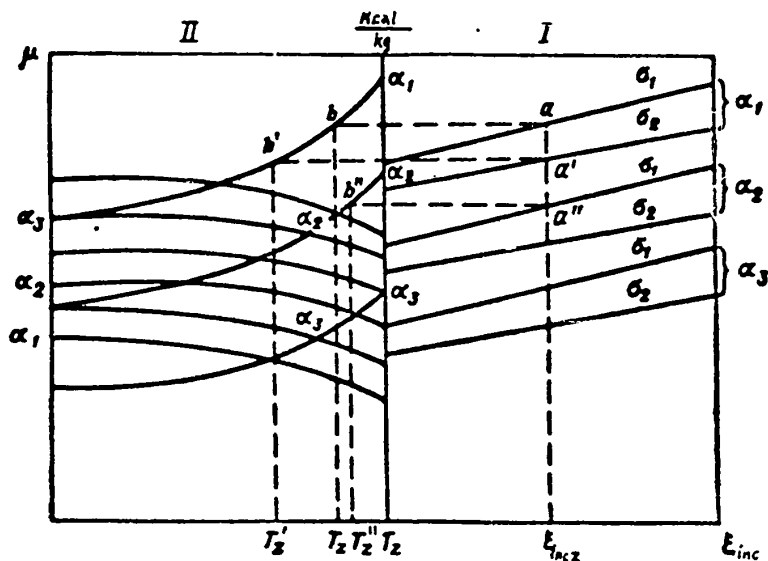


Fig. 4 - Nomogram for the Technique of Constructing the Diagrams

culate the combustion process, i.e., to determine the combustion temperature, and, as will be shown below, the molecular weight of the combustion products.

Let a propellant be burned in the combustion chamber at a coefficient of excess oxidizer  $\alpha_1$ . Let the concentration of oxidizer in this propellant be  $\sigma_1$ ; the experimental data determine the coefficient of heat liberation for this case to be  $\xi_{inc z}$ . On diagram I (Fig. 4) we find the point a, characterizing the combustion conditions of the propellant and, consequently, also the value of the quantity  $x_c + i_c - (1 - \xi_{inc z}) H_p$ . Since the value of  $i_z + x_z$  is equal to the value of  $x_c + i_c - (1 - \xi_{inc z}) H_p$ , then, after transferring the point a to diagram II, we find the point b at the intersection with the curve, corresponding to the specified value of the coefficient of excess oxidizer  $\alpha_1$ ; by projecting the point b onto the ab-

scissa, we find the corresponding combustion temperature. If the concentration has varied and has now become  $\alpha_2$ , then we find the point  $a'$  on the right part of the diagram and, by analogy, the point  $b'$  and the temperature  $T'_z$  on the left part of the diagram. If, at the former concentration, the coefficient of excess oxidizer has increased to the value  $\alpha_2$ , then the corresponding construction will give the point  $a''$  on the right side of the diagram, and on the left side the point  $b''$  and the combustion temperature  $T''_z$ . For calculating the combustion chamber, we must also know the parameters of the combustion products; for this purpose, we plot on diagram II the curves of the molecular weight of the gases under various combustion conditions.

The part of the computational diagram given in Fig.4 permits determining the state of the combustion products before their entrance to the nozzle, i.e., before the beginning of discharge.

The calculation of the process of efflux is based on eq.(58):

$$i_a + x_a + A \frac{w_a^2}{2g} = x_c + i_c - (1 - \xi_{inc a}) H_p$$

or

$$e_a + A \frac{w_a^2}{2g} = e_c - (1 - \xi_{inc a}) H_p$$

This equation takes account of the partial afterburning of the propellant in the nozzle, since the value of the coefficient of heat liberation in the cross section of the nozzle,  $\xi_{inc a}$ , is taken as a greater value than at the exit of the combustion chamber  $\xi_{inc z}$ . The value of the right side of the equation is determined from diagram I for a known value of  $\xi_{inc a}$ .

To construct the diagram III, permitting us to calculate the efflux, we must know the value of the left side of the equation. The values of  $i_a$  and  $x_a$  are determined for various temperatures by the equations:

$$x_a = \frac{1}{\mu_a p_a} \sum p_{ia} X_{i,i}$$

$$i_a = \frac{1}{\mu_a p_a} \sum p_{ia} I_{i,i}$$

or, using Appendix II, by the formula

$$e_a = \frac{1}{\mu_a p_a} \sum p_{ia} E_{ia}$$

The value of  $A \frac{w_a^2}{2g}$  is determined in a somewhat more complicated way, by the aid of eq.(63), as follows:

$$A \frac{w_a^2}{2g} = A \frac{n}{n-1} RT_z \left[ 1 - \left( \frac{p_a}{p_z} \right)^{\frac{n-1}{n}} \right],$$

where, according to eq.(64),

$$n = \frac{\lg \frac{p_z}{p_a}}{\lg \frac{p_z}{p_a} \cdot \frac{T_a}{T_z}}.$$

Thus the value of  $A \frac{w_a^2}{2g}$  is determined both for the final temperature  $T_a$  and for the initial temperature  $T_z$ .

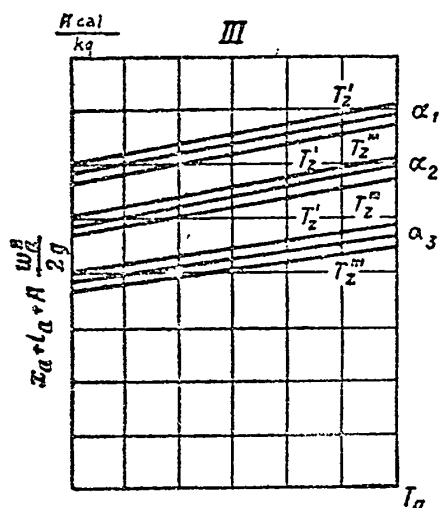
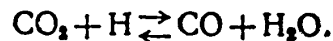


Fig.5 - For the Technique of  
Calculating the Efflux of the  
Combustion Products

To determine the values of  $i_a$  and  $x_a$  or the values of  $e_a = i_a + x_a$  with the efflux in a state of complete equilibrium, we must know the composition of the combustion products, which may be calculated from the above system of equations. However, due to the considerable temperature drop on expansion in the nozzle, the combustion products, on emergence, will have a considerably simpler gas composition: they will contain only  $\text{CO}_2$ ,  $\text{CO}$ ,  $\text{H}_2\text{O}$ ,  $\text{H}_2$ , and  $\text{N}_2$ . Thus the system of equations is simplified, and only five equations remain.

The chemical equilibrium will be characterized

only by the water-gas reaction which takes place without change in the number of molecules:



The equation of chemical equilibrium for this case is found by dividing eq.(36) by eq.(37):

$$\frac{P_{\text{CO}} \cdot P_{\text{H}_2\text{O}}}{P_{\text{CO}_2} \cdot P_{\text{H}_2}} = \frac{K_{p_1}}{K_{p_2}}.$$

Consequently, the system of equations reduces to the form of the system of eqs.(65) - (69).

We can then construct on the diagram III, the values of  $e_a + A \frac{w_a^2}{2g}$  for any temperatures. Rough calculation show that a family of straight lines is obtained on diagram III. Figure 5 shows the general form of this diagram.

On this diagram, we have  $\alpha_3 > \alpha_2 > \alpha_1$  and  $T_z' > T_z'' > T_z'''$ . After combining diagrams I and III, we get that part of the diagram which permits determination of the final stage of the combustion products with extreme equilibrium efflux.

This part of the diagram is shown in Fig.6.

If the point a, on the left part of the diagram, determines the state of the gases before they enter the nozzle, and if, in finding this point, the additional quantity of heat liberated during efflux as a result of afterburning of the propellant was taken into consideration, then, projecting it on the right side of the diagram to the intersection with the straight line characterized by the specified value of  $\alpha$  and of the temperature  $T_z$ , found earlier on diagram II, we find the point b; by then projecting this point on the abscissa, we determine the temperature of the gases at emergence from the nozzle under extreme equilibrium conditions of discharge.

To determine the exhaust velocity under the conditions found, diagram IV is placed under diagram III. On diagram IV are plotted the curves determining the exhaust velocity for various values of  $T_z$  and  $T_a$ . The exhaust velocity depends on the initial and final temperatures of efflux and also on the coefficient of excess oxidizer, and for this reason a very large number of curves would be obtained on dia-



gram IV, thus making calculations from it exceedingly difficult. To eliminate this, curves of adjacent values are combined, so as to considerably reduce the number of curves, making calculation entirely possible with an error of not more than 8 m/sec

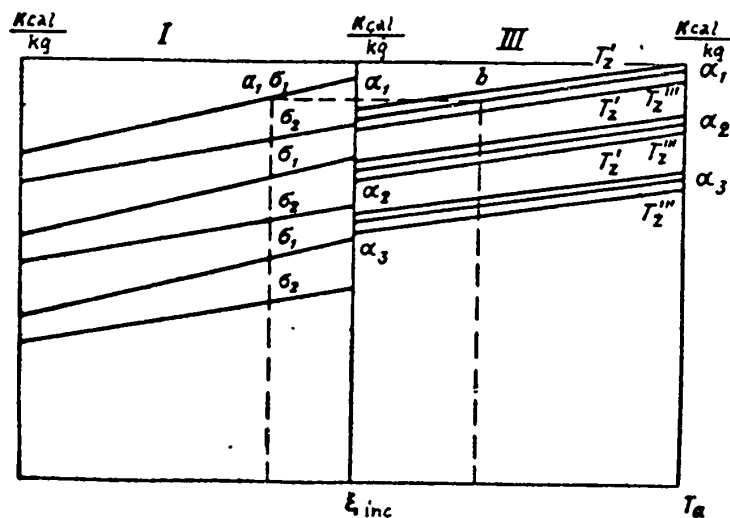


Fig.6 - Determination of the Final State of the Combustion Products

in determining the velocity; a Table indicating the number of the curve that must be used in one case or the other is, in this case, annexed to this part of the diagram.

Diagrams I - IV make it possible to calculate the extreme equilibrium efflux. The rest of the calculation is performed by the usual formulas.

The calculation procedure from the diagram (Fig.7) is as follows: The quantities specified are: the coefficient of excess oxidizer  $\alpha$ , the concentration of the oxidizer  $\sigma$ , the assumed coefficient of heat liberation in the combustion chamber  $\xi_{inc z}$ , and the coefficient of heat liberation at the end of the nozzle  $\xi_{inc a}$ . From these data, in the central part of diagram I, we find point 1 corresponding to  $\xi_{inc z}$ , and from it, on the curve of  $x_c + i_c - (1 - \xi_{inc z}) H_{up}$  we find the point 2 for given values of  $\alpha$  and  $\sigma$ ; the point 2 is projected on the left part of diagram II, to the intersection with the curve of  $i_z + x_z$  for the assigned  $\alpha$  at point 3. Transferring point 3 to the abscissa, we find the combustion temperature  $T_z$  under the specified combustion conditions (point 4), and from the curves

for  $\mu$  we determine the molecular weight of the combustion products; this weight is defined by point 5. To calculate the process of efflux, we turn to diagram I and find on the abscissa the point 7 corresponding to the specified value of  $\xi_{inc a}$ ; from this point we must find on the curve, for given values of  $\alpha$  and  $\sigma$ , the new point 8, characterizing the state of the combustion products at the end of the nozzle, i.e., at the coefficient of heat liberation  $\xi_{inc a}$ . Point 8 is projected on the right part of diagram III, and at the intersection with the curve for the as-

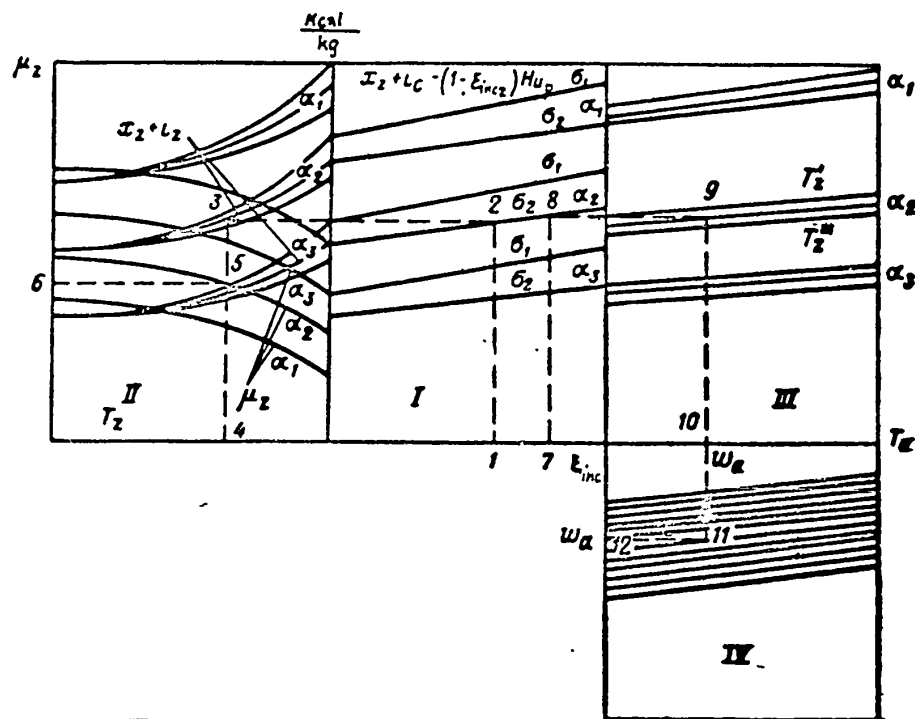
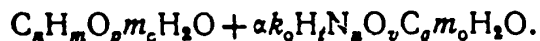


Fig.7 - Superimposed Diagram for Calculating the Processes of Combustion and Efflux in a Liquid-Propellant Rocket Engine

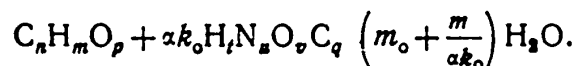
signed  $\alpha$  and the value found for  $T_z$ , we find the point 9; the point 9 is transferred to the abscissa axis, and the temperature of the gases on exit from the nozzle,  $T_a$ , is found at the point 10. Continuing the projection further on diagram IV to intersection with the velocity curve constructed for a specified value of  $\sigma$  and the obtained temperature  $T_z$ , we find the point 11, determining the exhaust velocity of the

gases under extreme equilibrium efflux.

In the case of a lowered concentration of both components, the entire water content may be attributed to the oxidizer for the construction of the diagram, which involves no changes in the calculation. Let the propellant be



The total quantity of water contained in this propellant is  $(m_c + \alpha k_o m_o)$  moles. Attributing all this quantity of water to the oxidizer, for calculation by the diagram we take the propellant as



In calculating and constructing a diagram by this method, we use the constant pressure  $p_z$  at the entrance to the nozzle and the constant pressure  $p_a$  at the exit from it; thus the diagrams III and IV are calculated for a definite pressure drop. Rough calculations show that, owing to the small influence of the pressure on the degree of dissociation, even a substantial change in pressure will have almost no effect on the composition of the combustion products; therefore, the diagram may be used for other pressure drops as well, determining the actual values by interpolation. We may also utilize one and the same pressure drop for calculations at other pressures, but with the same pressure drop, i.e., if a pressure of 32 atm abs in the combustion chamber is noted, and a pressure of 0.8 atm abs is noted at the exit, then diagrams III and IV may be used for calculation at the initial pressure of 40 atm abs and at the pressure of 1 atm abs at the exit.

The influence of the pressure drop on the velocity is considerable, and a calculation must always be performed at a specified pressure drop.

### 3. Diagram of the Propellant Kerosene + Nitric Acid

#### (Appendix V)

The diagram for the propellant kerosene + nitric acid is constructed from the

following principal data:

The heat value of kerosene is taken as 10,275 kcal/kg; nitric acid of 96% concentration is taken as the principal oxidizer, and additional calculations are given for nitric acid of 92 and 100% concentrations. The coefficient of excess oxidizer is taken from  $\alpha = 0.7$  to  $\alpha = 0.9$ . On diagram I, the straight lines are plotted giving the values of  $[x_c + i_c - (1 - \xi_{inc} z) H_p]$  in the range of values of the coefficient of heat liberation from  $\xi_{inc} z = 0.7$  to  $\xi_{inc} z = 1$ .

The diagram is constructed under the assumption that, on exit from the nozzle, the gas pressure is  $p_a = 1$  atm abs; for the pressure in the combustion chamber three versions have been taken:  $p_z = 25$  atm abs,  $p_z = 50$  atm abs and  $p_z = 100$  atm abs; owing to this fact, the curves for all three pressures have been plotted on diagram II, since the pressure in the combustion chamber in this case has a perceptible effect on the degree of dissociation and, consequently, also on the molecular weight of the combustion products and on their energy content  $e_z = i_z + x_z$ .

Diagram III is given in three versions, just as diagram IV is given for all three processes taken for the calculation.

On diagram III are plotted families of straight lines characterizing the state of the combustion products at the exit from the nozzle, for various combustion temperatures and for various values of the coefficient of excess oxidizer.

On diagram IV are plotted only the curves of the exhaust velocity, and, as stated above, the originally very large number of curves has been reduced to 22 - 23 by combining the curves for which the exhaust velocity differs by no more than 7 - 8 m/sec; Tables are annexed to this diagram indicating, for given efflux conditions ( $\alpha$  and  $T_z$ ), the number of the curve that must be used.

We shall now give examples of calculating the processes in a liquid-propellant rocket engine from this diagram.

Example 7. Calculate the combustion processes and the processes of efflux of the combustion products for the following propellant: kerosene + 96% nitric acid

at a coefficient of excess oxidizer  $\alpha = 0.8$ ; the coefficient of heat liberation in the combustion chamber is  $\xi_{inc z} = 0.92$ , and at the exit from the nozzle it is  $\xi_{inc a} = 0.98$ ; the pressure in the combustion chamber is 25 atm abs, and at the exit from the nozzle it is 1 atm abs.

We find on the abscissa of diagram I the point o, corresponding to the value  $\xi_{inc z} = 0.92$ , and project it, in the vertical, to intersection with the straight line for  $\alpha = 0.8$  and  $\sigma_o = 96\%$ ; at this intersection we find the point a; this point we transfer, parallel to the abscissas, to the intersection on diagram II with the curve for  $\alpha = 0.8$  and  $p_z = 25 \text{ kg/cm}^2$ ; at this intersection we find the point b, and by projecting this point onto the abscissa we determine the combustion temperature  $T_z = 2800^\circ \text{ abs}$ ; the point of intersection of the ordinate of the point b with the curve of molecular weight for  $\alpha = 0.8$  and  $p_z = 25 \text{ atm abs}$  is projected on the ordinate to find the molecular weight  $\mu_z = 24.56$ .

To obtain the data necessary for calculating the process of efflux, let us now return to diagram I, and on the abscissa let us find the new point c corresponding to the coefficient of heat liberation at the exit from the nozzle  $\xi_{inc a} = 0.98$ , and, projecting it upward along the vertical to the intersection with the straight line  $\alpha = 0.8$ ,  $\sigma_o = 96\%$ , we find the point e; by transferring this point to the diagram III to the intersection with the straight line corresponding to temperature  $T_z = 2800^\circ \text{ abs}$  and  $\alpha = 0.8$ , we get the point k characterizing the state of the combustion product at the end of efflux; projecting this point on the abscissa, we find the temperature of the combustion product at the exit from the nozzle in the extreme equilibrium efflux to be  $T_a = 1825^\circ \text{ abs}$  (according to the calculation given in example 5, this temperature is  $1796^\circ \text{ abs}$ ); projecting this same point downward on diagram IV to intersection with the curve l2, corresponding (according to the annexed diagram) to  $\alpha = 0.8$  and  $T_z = 2800^\circ \text{ abs}$ , we find the rate of efflux for the extreme equilibrium efflux to be equal to  $w_a = 2225 \text{ m/sec}$  (from the calculation in example 5 this velocity was  $2229 \text{ m/sec}$ ).

Example 8. Calculate the combustion processes and the processes of efflux of the combustion products for the following propellant: kerosene + 96% nitric acid; the coefficient of excess oxidizer is taken as  $\alpha = 0.85$ ; the coefficient of heat liberation in the combustion chamber as  $\xi_{inc z} = 0.9$ , and the same coefficient at the exit from the nozzle as  $\xi_{inc a} = 0.97$ . The pressure in the combustion chamber is 40 atm abs, and in the cross section of the nozzle it is 1 atm abs.

Owing to the fact that the diagram has no pressure drop equal to the assigned pressure drop, we shall perform the calculation roughly, assuming linear relations between the quantities, which yields an accuracy sufficient, in practice, for small pressure changes.

Let us use the diagram for two parallel calculations: for the pressure in the combustion chamber  $p_z = 25$  atm abs and 50 atm abs. We find the following values from the diagram:

$p_z$ atm abs	25	50
$T_z$ abs	2800	2835
$\mu_z$	25,24	25,49
$T_a$ abs	1768	1570
$w_a$	2170	2335

By interpolation we find, for 40 atm abs:  $T_z = 2820^\circ$  abs,  $\mu_z = 25.27$ ,  $T_a = 1650^\circ$  abs,  $w_a = 2270$  m/sec,  $R_z = 33.56$ .

The value of the index of polytropy for 40 atm abs may be found. It is

For  $p_z = 25$  atm abs

$$n = \frac{\lg 25}{\lg 25 \frac{1768}{2800}} = 1,167,$$

For  $p_z = 50$

$$n = \frac{\lg 50}{\lg 50 \frac{1570}{2835}} = 1,178.$$

For a pressure of 40 atm abs, the value of the index of polytropy is found to be

$$n = 1,1736.$$

The temperature in the cross section of the nozzle in this case will be

$$T_a = 2820 \left( \frac{1}{40} \right)^{\frac{0.1736}{1.1736}} = 1634^\circ$$

Consequently, the exhaust velocity will be, according to this calculation,

$$w_a = \sqrt{2 \cdot 9.81 \frac{1.1736}{0.1736} 33.56 (2820 - 1634)} = 2298 \text{ m/sec.}$$

This value of the velocity will be more accurate.

Example 9. For the propellant kerosene + 96% nitric acid, find the coefficient of excess oxidizer at which the maximum specific thrust will be developed. The pressure in the combustion chamber is 50 atm abs, and at the exit from the nozzle it is 1 atm abs, we take  $\xi_{inc z} = 0.9$ ,  $\xi_{inc a} = 0.95$ .

For various values of  $\alpha$ , in accordance with the conditions of the problem, the diagram gives the following results:

$\alpha$	$T_z$ abs	$T_a$ abs	$w_a$ m/sec
0.7	2560	1365	2300
0.75	2685	1500	2350
0.8	2795	1590	2380
0.82	2820	1625	2392
0.85	2840	1685	2390
0.9	2870	1750	2385

Thus, under the specified conditions, the maximum exhaust velocity, and consequently also the maximum specific thrust, is obtained roughly at  $\alpha = 0.82$ .

For the same conditions, but at  $\xi_{inc z} = 1$ , we get the following data:

$\alpha$	$T_z$ abs	$T_a$ abs	$w_a$ m/sec
0.7	2780	1410	2385
0.75	2875	1550	2425
0.8	2945	1640	2450
0.85	2980	1720	2440
0.9	3005	1800	2420

Thus in this case the maximum specific thrust is found to be about  $\alpha = 0.8$ .

It should be borne in mind that, owing to the insufficient accuracy in constructing the first diagram and to the graphic determination of the quantities, the figures obtained can characterize the processes only qualitatively.

Example 10. For the data found in example 9, determine the specific propellant consumption, by weight and by volume, per kg of thrust per hour, and also determine its composition by weight and by volume.

In a case where we allow for the incompleteness of propellant combustion and for its afterburning in the nozzle, the specific consumption of propellant by weight,  $C_p$ , kg/kg hr is determined from the gas exhaust velocity found in example 9 at various values of  $\alpha$ :

$\alpha$	$w_a$ m/sec	$C_p$ kg/kg hr
0,70	2300	15,35
0,75	2350	15,03
0,80	2380	14,84
0,82	2392	14,76
0,85	2390	14,77
0,90	2385	14,81

The molecular weight (arbitrary) of kerosene is

$$\mu_c = 100,$$

and that of nitric acid is

$$\mu_o = \frac{63 \cdot 100}{96} = 65,625.$$

Consequently, the composition of the propellant by weight, depending on the coefficient of excess oxidizer  $\alpha$ , will be expressed by the formulas

$$g_c = \frac{100}{100 + 65,625\alpha k_o}; \quad g_o = \frac{65,625}{100 + 65,625\alpha k_o}$$

or, since in this case  $k_o = 8,48$ , then:

$$g_c = \frac{100}{100 + 556,5\alpha}; \quad g_o = \frac{556,5\alpha}{100 + 556,5\alpha}$$

After substituting the values of  $\alpha$ , the following results are obtained for the composition of the propellant by weight and for the specific consumption of the components in kg/kg hr:

$\alpha$	Composition by Weight in %		Specific Consumption of Components in kg/kg hr	
	$g_c$	$g_o$	$C_{p_c}$	$C_{p_o}$
0,70	20,4	79,6	3,13	12,22
0,75	19,3	80,7	2,90	12,13
0,80	18,3	81,7	2,72	12,12
0,82	18,0	82,0	2,66	12,10
0,85	17,4	82,6	2,57	12,20
0,90	16,6	83,4	2,46	12,35



The composition of the propellant by volume and its specific consumption by volume, per kg of thrust per hour, at the following specific gravities for the components:

$$\gamma_c = 0,82 \text{ kg/kg hr} \quad \gamma_o = 1,485 \text{ kg/ltr}$$

are found to be the following for the specified conditions:

$\alpha$	$v \text{ ltr/kg hr}$	$v_o \text{ ltr/kg hr}$	$v \text{ ltr/kg hr}$
0,70	3,82	8,23	12,05
0,75	3,54	8,17	11,71
0,80	3,32	8,16	11,48
0,82	3,24	8,15	11,39
0,85	3,15	8,22	11,37
0,90	3,00	8,32	11,32

For the case of the calculation at  $\xi_{inc} z = 1$ , we get the following values:

$\alpha$	$w_a \text{ m/sec}$	$C_p \text{ kg/kg hr}$
0,70	2385	14,81
0,75	2425	14,56
0,80	2450	14,41
0,85	2440	14,47
0,90	2420	14,59

Consequently, with the former composition of the propellant by weight, the specific consumption of the components in kg/kg hr will be as follows:

$\alpha$	$C_p \text{ kg/kg hr}$	$C_{p_o} \text{ kg/kg hr}$
0,70	3,02	11,79
0,75	2,81	11,75
0,80	2,64	11,77
0,85	2,52	11,95
0,90	2,42	12,17

The consumption of propellant by volume, and that of its components, is determined similarly:

$\alpha$	$v_c \text{ ltr/kg hr}$	$v_o \text{ ltr/kg hr}$	$v \text{ ltr/kg hr}$
0,70	3,69	7,94	11,63
0,75	3,43	7,92	11,35
0,80	3,22	7,93	11,15
0,85	3,07	8,05	11,12
0,90	2,95	8,19	11,14

#### 4. Diagram of the Propellant: Ethyl Alcohol + Hydrogen Peroxide (Appendix VI)

This diagram differs from the preceding in being simpler, owing to the con-

siderably lower combustion temperatures and, consequently, to the far lesser degree of dissociation.

The diagram is constructed on the following bases: The propellant used is ethyl alcohol of 70, 80, 90, and 100% concentration, and 80% hydrogen peroxide. The heat value of the components is taken according to the data given in Chapter I. The calculation has been performed for a coefficient of excess oxidizer ranging from  $\alpha = 0.8$  to  $\alpha = 1.0$ . On diagram I are plotted straight lines giving the values of  $[x_c + i_c - (1 - \xi_{inc\ z}) H_p]$  in the range of values of the coefficient of heat liberation from  $\xi_{inc\ z} = 0.8$  to  $\xi_{inc\ z} = 1.0$ .

The diagram has been calculated under the assumption that the pressure at the exit from the nozzle is  $p_a = 1$  atm abs, and three versions have been taken for the pressure in the combustion chamber:  $p_z = 25, 20,$  and  $15$  atm abs.

Diagram II gives the curves determining the values of  $Q_z = i_z + x_z$ , and curves giving the molecular weight of the combustion products. Owing to the low degree of dissociation, the curves of energy content differ very little from straight lines, and, in addition, within the limits of accuracy of the constructions, they yield the same values for the pressures assumed in the diagram: the curves for the molecular weight give a constant molecular weight up to temperatures of  $2100^\circ$  abs, and then, owing to a certain increase in the degree of dissociation, the molecular weight decreases somewhat.

Diagrams III and IV are given in three versions corresponding to the pressures in the combustion chamber adopted in the calculation.

Diagram III gives a family of straight lines determining the values of  $(e_a + \frac{1}{2} \frac{w_a^2}{g})$  for various values of  $\alpha$  and various combustion temperatures  $T_z$ .

Diagram IV, besides the curve determining the gas exhaust velocity  $w_a$ , also gives curves permitting a determination of the index of polytropy for discharge under conditions of complete equilibrium.

Examples of the calculations by this diagram are given below.

Example 11. Calculate the processes in the combustion chamber and nozzle for the propellant: 80% ethyl alcohol and 80% hydrogen peroxide. The coefficient of excess oxidizer is 0.85.

The coefficients of heat liberation adopted are: for the exit of the combustion chamber  $\xi_{inc z} = 0.85$ , for the exit cross section of the nozzle  $\xi_{inc a} = 0.95$ . The calculation is also to be performed at  $\xi_{inc z} = 1$ .

The pressure in the combustion chamber is  $p_z = 25$  atm abs, and at the exit from the nozzle it is  $p_a = 1$  atm abs.

On the abscissa of diagram I we find the points corresponding to the specified coefficient of heat liberation  $\xi_{inc z} = 0.85$ , projected upward to intersection with the straight line characterizing the composition of the fuel ( $\alpha = 0.85$ ,  $\sigma_c = 80\%$ ); the point so obtained is then transferred parallel to the abscissa, to intersection on diagram II with the curve  $\alpha = 0.85$ ; this curve, projected on the abscissa, determines the combustion temperature, which is  $1996^\circ$  abs. To determine the molecular weight of the combustion products, the point obtained on diagram II is transferred to the curve  $\mu_z$  corresponding to  $\alpha = 0.85$  and  $\sigma_c = 80\%$ , and we find  $\mu_z = 20.86$ .

To calculate the process of efflux, let us return to the diagram I and find on the abscissa the point corresponding to the coefficient of heat liberation at the exit from the nozzle  $\xi_{inc z} = 0.95$ ; projecting this point upward to intersect the straight line  $\alpha = 0.85$  and  $\sigma_c = 80\%$ , and transferring the point so obtained to diagram III to intersection with the straight line corresponding to the specified coefficient of excess oxidizer 0.85 and to the combustion temperature found,  $T_z = 1996^\circ$  abs (the  $2000^\circ$  abs curve), we find the point characterizing the state of the combustion products at the exit from the nozzle; transferring this point to the abscissa, we find the gas temperature at the exit from the nozzle under extreme equilibrium flow, namely  $T_a = 1295^\circ$  abs; then, on diagram IV, at the intersection with the curve of velocities 8, corresponding (according to the Table) to the assigned conditions, we determine the exhaust velocity to be  $w_a = 2055$  m/sec; simultaneously,

the intersection with the curve of the values of the index of polytropy, corresponding to the combustion temperature  $1996^{\circ}$  abs found, gives the value of the index of polytropy for the extreme equilibrium efflux,  $n = 1.156$ .

These data permit us to calculate the exhaust nozzle.

For the second case, when the calculation is performed without allowing for the incomplete combustion of the propellant due to incomplete mixing ( $\xi_{inc z} = 1$ ), the following values are found from the diagram. For  $\alpha = 0.85$  and  $\sigma_c = 80\%$ , at  $\xi_{inc z} = 1$  (on the ordinate), we determine the energy content of the propellant and, consequently, also the energy content of the combustion products before entering the nozzle,  $e_c = e_z = 1196$  kcal/kg; on the intersection of the line  $e_z = \text{const}$ , on diagram II, with the curve  $\alpha = 0.85$ , we get a point determining the combustion temperature  $T_z = 2245^{\circ}$  abs, and the transfer of this point to the curve  $\mu_z = \text{const}$  determines the molecular weight of the combustion products, which is equal to 20.55.

Consequently, the gas constant is  $R_z = 41.26$ . At the intersection of the same line  $e_z = \text{const}$  with the line  $T_z = 2245^{\circ}$  abs and  $\alpha = 0.85$ , from diagram III (at  $p_z = 25$  atm abs) we get the point determining a gas temperature at the nozzle exit of  $T_a = 1315^{\circ}$  abs.

This point, projected on diagram IV to the intersection with the velocity curve 13, determines an exhaust velocity equal to 2120 m/sec; when projected to the intersection with the curve of the index of polytropy for  $T_z = 2245^{\circ}$  abs, we find the index of polytropy to be  $n = 1.2$ . A calculation of the exhaust velocity, from the value of the index of polytropy so found, gives

$$w_s = \sqrt{2 \cdot 9,81 \frac{1,2}{0,2} 41,26 (2245 - 1315)} = 2124 \text{ m/sec}$$

The discrepancy with the velocity determined by the diagram is small.

Example 12. Trace, on the diagram, the influence of the concentration of ethyl alcohol on the processes in the liquid-propellant rocket engine.

Assume that  $\alpha = 0.8$ ,  $\xi_{inc z} = 0.95$ ,  $\xi_{inc a} = 1$ ,  $p_z = 20$  atm abs.

If the above-indicated constructions are drawn on the diagram, we get the following results:

$s_c \%$	$q_s \text{ Kcal/Kg}$	$T_s \text{ abs}$	$\mu_s$	$T_g \text{ abs}$	$n$	$w_a \text{ m/sec}$
100	1280	2238	20,75	1360	1,200	2075
90	1242	2177	20,65	1320	1,203	2040
80	1199	2108	20,55	1270	1,205	1995
70	1143	2020	20,45	1210	1,207	1950

Thus, decreasing the concentration of alcohol lowers the temperature of all processes both in the combustion chamber and in the nozzle, lowers the exhaust velocity, decreases the molecular weight (owing to the increased quantity of water vapor), and increases the index of polytropy.

##### 5. Computational Diagrams for $\xi_{inc} z = 1$

Computational diagrams constructed under the assumption that the incomplete combustion of the propellant is determined only by the dissociation of the combustion products and a value of  $\alpha$  less than unity, i.e., at  $\xi_{inc} z = 1$ , differ from the above described diagrams by being considerably simpler. Today it is very likely that they may find far more frequent practical application, since we still have no more or less reliable data on the actual values of the coefficients of heat liberation due to incomplete mixing of the components.

The diagram is constructed for a definite fuel at various values of the coefficient of excess oxygen  $\alpha$  and for various pressures in the combustion chamber; the pressure at the exit from the nozzle is usually taken as 1 atm abs. The diagram is divided by the abscissa into two parts, an upper and a lower. The upper part of the diagram serves to find the quantities characterizing the state of the combustion products at the exit of the combustion chamber, and the lower for the quantities characterizing the state of the gases at the nozzle exit. Thus the principal quantities for the combustion process in the combustion chamber can be found on the upper part of the diagram, and the principal quantities for the process of efflux from the nozzle may be found on the lower part.

On the abscissa we plot the values of the coefficient of excess oxidizer  $\alpha$ ; on the upper part of the diagram, the temperature curves, permitting the combustion temperature to be determined at various values of  $\alpha$  for a definite pressure; the curves move upward as the coefficient  $\alpha$  increases, since in this case the completeness of combustion also increases. For each pressure in the combustion chamber there is a separate curve. The temperature curve is located higher on the diagram the greater the pressure  $p_z$  in the combustion chamber, since the rise in pressure decreases the degree of dissociation, and thus increases the completeness of combustion of the propellant. Further than this, on this part of the diagram we also plot the curves of the values of the gas constant of the combustion products; these curves decline with increasing values of  $\alpha$ , owing to the more complete combustion and the increase in the quantity of diatomic gases; owing to the decrease in the degree of dissociation with increasing pressure, the curves of the gas constant for higher pressures are located lower. The temperature scale is placed on the left ordinate and the scale of the gas constant on the right.

On the lower part of the diagram, curves of the gas temperatures at nozzle exit are constructed; these curves rise with increasing coefficient of excess oxidizer, since, at one and the same pressure drop, the exit temperature is higher, the higher the temperature in the combustion chamber; the curves of the temperature  $T_a$  for higher pressures in the combustion chamber are located lower, since the increase in the pressure drop lowers the exit temperature. The principal computational curve is the curve of the exit velocity of the combustion products  $w_a$ . These curves are located higher on the diagram, the greater the pressure drop used in the liquid-propellant rocket engine, i.e., the greater the pressure in the combustion chamber. On this same part of the diagram the curves of the gas constant at the nozzle exit,  $R_a$ , may be plotted.

Appendix VII gives a diagram for the propellant kerosene + 96% nitric acid.

We give below an example of a calculation by this diagram.

Example 13. Determine from the diagram (see Appendix VII) the effect of pressure in the combustion chamber on the specific consumption of the propellant kerosene + 96% nitric acid at  $\alpha = 0.75$  and  $\alpha = 0.85$ .

From the diagrams for the specified values of the coefficient of excess oxidizer, we find:

$p_s^* \text{ atm abs}$	$\alpha=0,75$			$\alpha=0,85$		
	$T_s^* \text{ abs}$	$T_a^* \text{ abs}$	$w_a \text{ m/sec}$	$T_s^* \text{ abs}$	$T_a^* \text{ abs}$	$w_a \text{ m/sec}$
25	2850	1730	2240	2930	1960	2260
35	2860	1650	2306	2952	1875	2334
50	2870	1530	2420	2985	1750	2437
75	2885	1440	2492	3005	1650	2510
100	2900	1365	2552	3032	1565	2568

Consequently the specific propellant consumption in kg per kg of thrust per hour will be as follows:

$p_s \text{ atm abs}$	$C_p \text{ kg/kg hr}$	
	$\alpha = 0,75$	$\alpha = 0,85$
25	15,77	15,63
35	15,31	15,13
50	14,59	14,49
75	14,17	14,07
100	13,84	13,75

Using the data on the composition of the fuel by weight obtained in example 10, the specific consumption of the components may be found; from the data of example 10 we have:

$$\begin{aligned} \text{for } \alpha = 0,75 \quad g_c &= 19,3\%, \quad g_o = 80,7\% \\ \text{for } \alpha = 0,85 \quad g_c &= 17,4\%, \quad g_o = 82,6\%. \end{aligned}$$

Calculations give the following specific consumption of the components in kg/kg hr.

$p_x$ atm abs	$\alpha=0,75$		$\alpha=0,85$	
	$C_{p_c}$	$C_{p_o}$	$C_{p_c}$	$C_{p_o}$
25	3,04	12,73	2,72	12,91
35	2,95	12,36	2,63	12,50
50	2,82	11,77	2,52	11,97
75	2,73	11,44	2,45	11,62
100	2,67	11,17	2,39	11,36

The specific consumption of the components by volume at specific gravities of  $\gamma_c = 0.82$  kg/ltr and  $\gamma_o = 1.485$  kg/ltr is found to be as follows:

$p_x$ atm abs	$v$ / $\kappa$					
	$\alpha=0,75$			$\alpha=0,85$		
	$v_c$	$v_o$	$v$	$v_c$	$v_o$	$v$
25	3,71	8,58	12,29	3,32	8,70	12,02
35	3,60	8,32	11,92	3,21	8,41	11,62
50	3,44	7,93	11,37	3,07	8,05	11,12
75	3,33	7,71	11,04	2,99	7,83	10,82
100	3,26	7,52	10,78	2,92	7,65	10,57



A P P E N D I X E S

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## Appendix I

## Equilibrium Constants

$T^\circ \text{ abs}$	$K_{p_1} = \frac{p_{\text{CO}} p_{\text{O}_2}^{0.5}}{p_{\text{CO}_2}}$	$K_{p_2} = \frac{p_{\text{H}_2} p_{\text{O}_2}^{0.5}}{p_{\text{H}_2\text{O}}}$	$K_{p_3} = \frac{p_{\text{OH}} p_{\text{H}_2}^{0.5}}{p_{\text{H}_2\text{O}}}$	$K_{p_4} = \frac{p_{\text{H}}^2}{p_{\text{H}_2}}$	$K_{p_5} = \frac{p_{\text{O}}^2}{p_{\text{O}_2}}$	$K_{p_6} = \frac{p_{\text{NO}}}{p_{\text{N}_2}^{0.5} p_{\text{O}_2}^{0.5}}$	$\frac{K_{p_1}}{K_{p_2}} = \frac{p_{\text{CO}} p_{\text{H}_2\text{O}}}{p_{\text{CO}_2} p_{\text{H}_2}}$
600	$0,8624 \cdot 10^{-20}$	$0,2333 \cdot 10^{-18}$	$0,3400 \cdot 10^{-21}$	$0,2153 \cdot 10^{-32}$	$0,1387 \cdot 10^{-36}$	$0,6039 \cdot 10^{-7}$	$0,3606 \cdot 10^{-1}$
700	$0,2900 \cdot 10^{-16}$	$0,2614 \cdot 10^{-15}$	$0,1265 \cdot 10^{-17}$	$0,6425 \cdot 10^{-27}$	$0,2240 \cdot 10^{-30}$	$0,8056 \cdot 10^{-6}$	0,1109
800	$0,1277 \cdot 10^{-13}$	$0,5156 \cdot 10^{-13}$	$0,6119 \cdot 10^{-15}$	$0,8426 \cdot 10^{-23}$	$0,1034 \cdot 10^{-25}$	$0,5624 \cdot 10^{-5}$	0,2475
900	$0,1445 \cdot 10^{-11}$	$0,3185 \cdot 10^{-11}$	$0,7568 \cdot 10^{-13}$	$0,1369 \cdot 10^{-19}$	$0,4450 \cdot 10^{-22}$	$0,8544 \cdot 10^{-4}$	0,4537
1000	$0,6331 \cdot 10^{-10}$	$0,8728 \cdot 10^{-10}$	$0,3604 \cdot 10^{-11}$	$0,5148 \cdot 10^{-17}$	$0,3631 \cdot 10^{-19}$	$0,8544 \cdot 10^{-4}$	0,7254
1100	$0,1389 \cdot 10^{-8}$	$0,1314 \cdot 10^{-8}$	$0,8519 \cdot 10^{-10}$	$0,6676 \cdot 10^{-15}$	$0,8820 \cdot 10^{-17}$	$0,2299 \cdot 10^{-3}$	1,056
1200	$0,1814 \cdot 10^{-7}$	$0,1267 \cdot 10^{-7}$	$0,1193 \cdot 10^{-8}$	$0,3886 \cdot 10^{-13}$	$0,8630 \cdot 10^{-15}$	$0,5246 \cdot 10^{-3}$	1,432
1300	$0,1591 \cdot 10^{-6}$	$0,8648 \cdot 10^{-7}$	$0,1116 \cdot 10^{-7}$	$0,1220 \cdot 10^{-11}$	$0,4191 \cdot 10^{-13}$	$0,1054 \cdot 10^{-2}$	1,840
1400	$0,1020 \cdot 10^{-5}$	$0,4501 \cdot 10^{-6}$	$0,7603 \cdot 10^{-7}$	$0,2358 \cdot 10^{-10}$	$0,1173 \cdot 10^{-11}$	$0,1918 \cdot 10^{-2}$	2,266
1500	$0,5087 \cdot 10^{-5}$	$0,1885 \cdot 10^{-5}$	$0,4016 \cdot 10^{-6}$	$0,3087 \cdot 10^{-9}$	$0,2113 \cdot 10^{-10}$	$0,3222 \cdot 10^{-2}$	2,699
1600	$0,2070 \cdot 10^{-4}$	$0,6615 \cdot 10^{-5}$	$0,1726 \cdot 10^{-5}$	$0,2944 \cdot 10^{-8}$	$0,2657 \cdot 10^{-9}$	$0,5075 \cdot 10^{-2}$	3,130
1700	$0,7131 \cdot 10^{-4}$	$0,2005 \cdot 10^{-4}$	$0,6250 \cdot 10^{-5}$	$0,2162 \cdot 10^{-7}$	$0,2486 \cdot 10^{-8}$	$0,7575 \cdot 10^{-2}$	3,555
1800	$0,2134 \cdot 10^{-3}$	$0,5383 \cdot 10^{-4}$	$0,1964 \cdot 10^{-4}$	$0,1277 \cdot 10^{-6}$	$0,1819 \cdot 10^{-7}$	$0,1081 \cdot 10^{-1}$	3,967
1900	$0,5687 \cdot 10^{-3}$	$0,1303 \cdot 10^{-3}$	$0,5475 \cdot 10^{-4}$	$0,6267 \cdot 10^{-6}$	$0,1080 \cdot 10^{-6}$	$0,1488 \cdot 10^{-1}$	4,363
2000	$0,1371 \cdot 10^{-2}$	$0,2892 \cdot 10^{-3}$	$0,1378 \cdot 10^{-3}$	$0,2631 \cdot 10^{-5}$	$0,5376 \cdot 10^{-6}$	$0,1982 \cdot 10^{-1}$	4,741

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2100	$0,3035 \cdot 10^{-2}$	$0,5954 \cdot 10^{-3}$	$0,3178 \cdot 10^{-3}$	$0,9658 \cdot 10^{-5}$	$0,2299 \cdot 10^{-5}$	$0,2568 \cdot 10^{-1}$	5,097
2200	$0,6240 \cdot 10^{-2}$	$0,1149 \cdot 10^{-2}$	$0,6797 \cdot 10^{-3}$	$0,3155 \cdot 10^{-4}$	$0,8624 \cdot 10^{-5}$	$0,3251 \cdot 10^{-1}$	5,433
2300	$0,1203 \cdot 10^{-1}$	$0,2094 \cdot 10^{-2}$	$0,1361 \cdot 10^{-2}$	$0,9313 \cdot 10^{-4}$	$0,2885 \cdot 10^{-4}$	$0,4031 \cdot 10^{-1}$	5,756
2400	$0,2195 \cdot 10^{-1}$	$0,3634 \cdot 10^{-2}$	$0,2573 \cdot 10^{-2}$	$0,2516 \cdot 10^{-3}$	$0,8738 \cdot 10^{-4}$	$0,4909 \cdot 10^{-1}$	6,089
2500	$0,3817 \cdot 10^{-1}$	$0,6037 \cdot 10^{-2}$	$0,4685 \cdot 10^{-2}$	$0,6284 \cdot 10^{-3}$	$0,2423 \cdot 10^{-3}$	$0,5884 \cdot 10^{-1}$	6,311
2600	$0,6333 \cdot 10^{-1}$	$0,9649 \cdot 10^{-2}$	$0,7947 \cdot 10^{-2}$	$0,1464 \cdot 10^{-2}$	$0,6215 \cdot 10^{-3}$	$0,6956 \cdot 10^{-1}$	6,563
2700	0,1013	$0,1490 \cdot 10^{-1}$	$0,1312 \cdot 10^{-1}$	$0,3207 \cdot 10^{-2}$	$0,1487 \cdot 10^{-1}$	$0,8119 \cdot 10^{-1}$	6,794
2800	0,1565	$0,2233 \cdot 10^{-1}$	$0,2091 \cdot 10^{-1}$	$0,6649 \cdot 10^{-2}$	$0,3345 \cdot 10^{-2}$	$0,9374 \cdot 10^{-1}$	7,008
2900	0,2344	$0,3256 \cdot 10^{-1}$	$0,3228 \cdot 10^{-1}$	$0,1312 \cdot 10^{-1}$	$0,7117 \cdot 10^{-2}$	0,1072	7,201
3000	0,3417	$0,4628 \cdot 10^{-1}$	$0,4841 \cdot 10^{-1}$	$0,2475 \cdot 10^{-1}$	$0,1441 \cdot 10^{-1}$	0,1213	7,382
3100	0,4854	$0,6436 \cdot 10^{-1}$	$0,7074 \cdot 10^{-1}$	$0,4485 \cdot 10^{-1}$	$0,2786 \cdot 10^{-1}$	0,1463	7,543
3200	0,6744	$0,8770 \cdot 10^{-1}$	0,1009	$0,7836 \cdot 10^{-1}$	$0,5174 \cdot 10^{-1}$	0,1520	7,690
3300	0,9179	0,1173	0,1410	0,1324	$0,9253 \cdot 10^{-1}$	0,1684	7,821
3400	1,226	0,1544	0,1933	0,2170	$0,9253 \cdot 10^{-1}$	0,1684	7,821
3500	1,610	0,2000	0,2601	0,3459	0,2680	0,2029	8,048
3600	2,081	0,2551	0,3444	0,5374	0,4364	0,2210	8,143
3700	2,652	0,3222	0,4492	0,8156	0,6926	0,2395	8,228
3800	3,334	0,4017	0,5780	1,212	1,073	0,2584	8,299
3900	4,141	0,4951	0,7343	1,763	1,624	0,2778	8,364
4000	5,087	0,6042	0,9217	2,519	2,408	0,2975	8,418

## Appendix II

## Energy Content

$$E = (I + X) \cdot 10^{-3} \text{ kcal/mole}$$

$T^{\circ} \text{ abs}$	$H_2$	$O_2$	$N_2$	$CO$	$OH$	$NO$	$CO_2$	$H_2O$	$H$	$O$
298	57,80	0	0	67,64	38,95	21,60	0	0	80,99	59,16
600	59,90	2,209	2,126	69,77	41,09	23,79	3,034	2,514	82,49	60,70
700	60,61	2,987	2,853	70,51	41,79	24,54	4,243	3,404	82,99	61,20
800	61,31	3,785	3,596	71,26	42,51	25,32	5,451	4,322	83,48	61,71
900	62,02	4,599	4,356	72,09	43,22	26,11	6,702	5,267	83,98	62,21
1000	62,74	5,426	5,130	72,82	43,95	26,91	7,984	6,240	84,48	62,71
1100	63,47	6,265	5,918	73,62	44,69	27,73	9,297	7,241	84,97	63,21
1200	64,20	7,114	6,710	74,43	45,44	28,56	10,63	8,268	85,47	63,71
1300	64,95	7,970	7,530	75,25	46,20	29,40	11,99	9,322	85,97	64,20
1400	65,71	8,834	8,351	76,08	46,97	30,24	13,37	10,40	86,46	64,70
1500	66,47	9,705	9,180	76,92	47,76	31,10	14,76	11,50	86,96	65,20
1600	67,25	10,58	10,02	77,77	48,55	31,96	16,17	12,63	87,46	65,70
1700	68,04	11,46	10,86	78,62	49,35	32,82	17,58	13,77	87,95	66,20
1800	68,83	12,35	11,71	79,47	50,16	33,69	19,01	14,93	88,45	66,70
1900	69,64	13,25	12,56	80,33	50,98	34,56	20,45	16,11	88,95	67,19
2000	70,46	14,15	13,42	81,70	51,80	35,44	21,90	17,30	89,44	67,69
2100	71,28	15,05	14,28	82,07	52,64	36,31	23,35	18,51	89,94	68,19
2200	72,11	15,96	15,15	82,94	53,48	37,20	24,81	19,73	90,44	68,69
2300	72,95	16,88	16,02	83,81	54,33	38,08	26,28	20,96	90,93	69,18
2400	73,80	17,80	16,89	84,69	55,18	38,96	27,75	22,20	91,43	69,68
2500	74,65	18,73	17,76	85,57	56,04	39,85	29,23	23,46	91,93	70,18
2600	75,51	19,66	18,64	86,45	56,91	40,74	30,71	24,72	92,43	70,68
2700	76,37	20,60	19,52	87,33	57,78	41,64	32,20	25,98	92,92	71,18
2800	77,25	21,54	20,40	88,22	58,65	42,53	33,70	27,26	93,42	71,68
2900	78,12	22,49	21,28	89,11	59,53	43,43	35,20	28,54	93,92	72,17
3000	79,01	23,45	22,17	90,00	60,41	44,32	36,70	29,83	94,41	72,68
3100	79,90	24,40	23,06	90,89	61,29	45,22	38,20	31,12	94,91	73,18
3200	80,79	25,37	23,94	91,78	62,18	46,12	39,71	32,42	95,41	73,68
3300	81,69	26,33	24,84	92,67	63,08	47,02	41,23	33,72	95,90	74,18
3400	82,59	27,30	25,73	93,57	63,97	47,93	42,74	35,03	96,40	74,69
3500	83,50	28,28	26,62	94,46	64,80	48,83	44,26	36,35	96,90	75,19
3600	84,41	29,26	27,51	95,36	65,78	49,74	45,79	37,66	97,39	75,69
3700	85,33	30,24	28,41	96,26	66,68	50,64	47,31	38,98	97,89	76,20
3800	86,26	31,22	29,30	97,16	67,59	51,55	48,84	40,30	98,39	76,70
3900	87,18	32,21	30,20	98,06	68,50	52,46	50,37	41,63	98,88	77,21
4000	88,12	33,20	31,10	98,96	69,42	53,37	51,91	42,96	99,38	77,72

## Appendix III

## Energy Content of Certain Propellant Components

$$e = i + x \text{ kcal/kg}$$

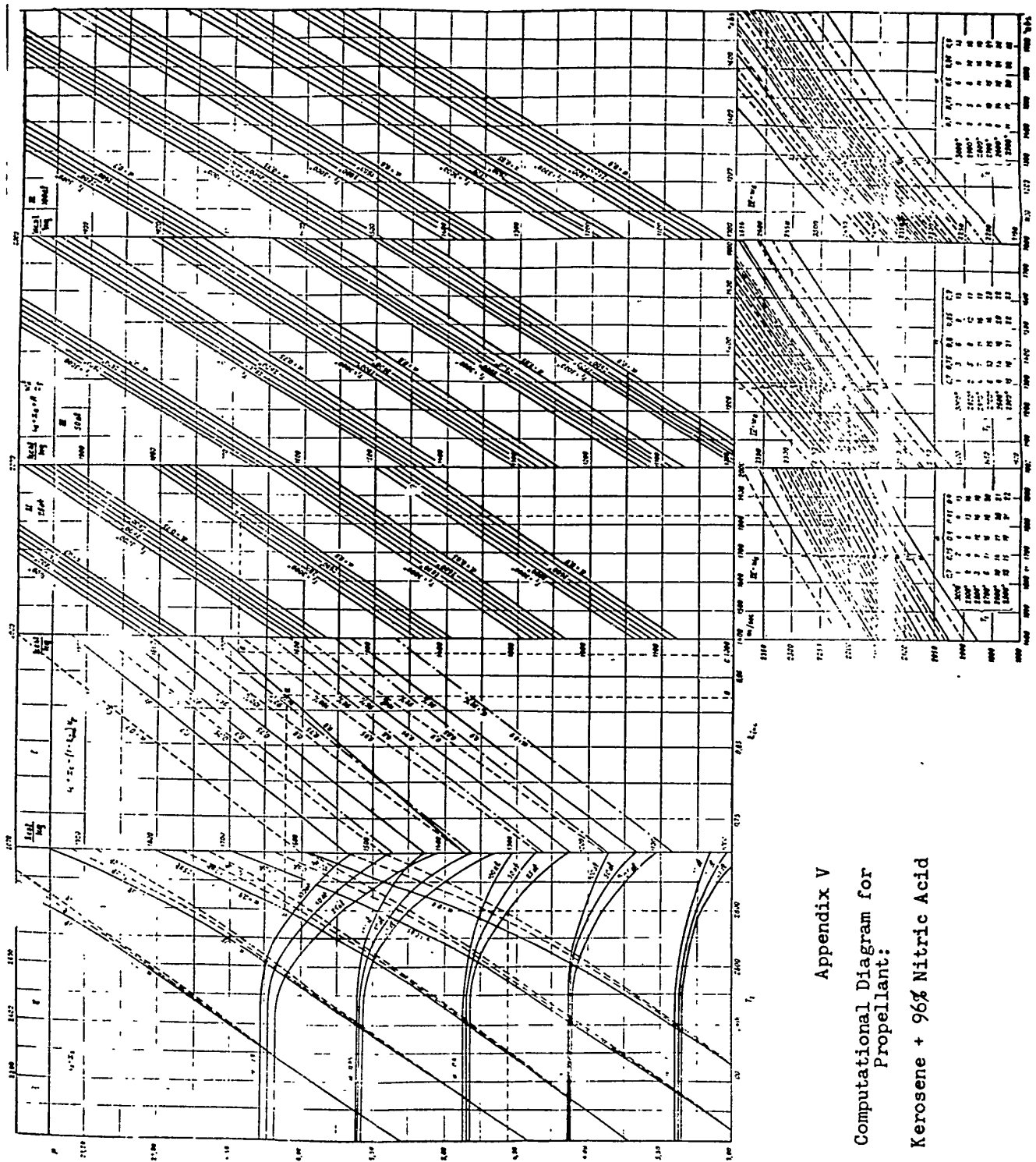
Substance	Chemical Formula	Molecular Wt. in kg/mole	e	At Absolute Temperature of
Ethyl alcohol, 100%	$C_2H_5OH$	46	6406	298,16°
Methyl alcohol, 100%	$CH_3OH$	32	4763	298,16°
Kerosene	$C_{12}H_{26}$ (Arbitr.)	100 (Arbitr.)	10200	298,16°
Triethylamine	$(C_2H_5)_3N$	101	9442	298,16°
Xylidene	$(CH_3)_2C_6H_4NH_2$	121	8451	298,16°
Toluene	$C_7H_8$	92	9685	298,16°
Hydrazine hydrate	$(NH_2)_2H_2O$	50	2306	298,16°
Hydrogen peroxide, 100%	$H_2O_2$	34	381	298,16°
Nitric acid, 100%	$HNO_3$	63	-198	298,16°
Tetranitromethane	$C(NO_2)_4$	196	525	298,16°
Oxygen (liquid)	$O_2$	32	-96	90,16°
Nitrogen tetroxide	$N_2O_4$	92	-75	294,31°

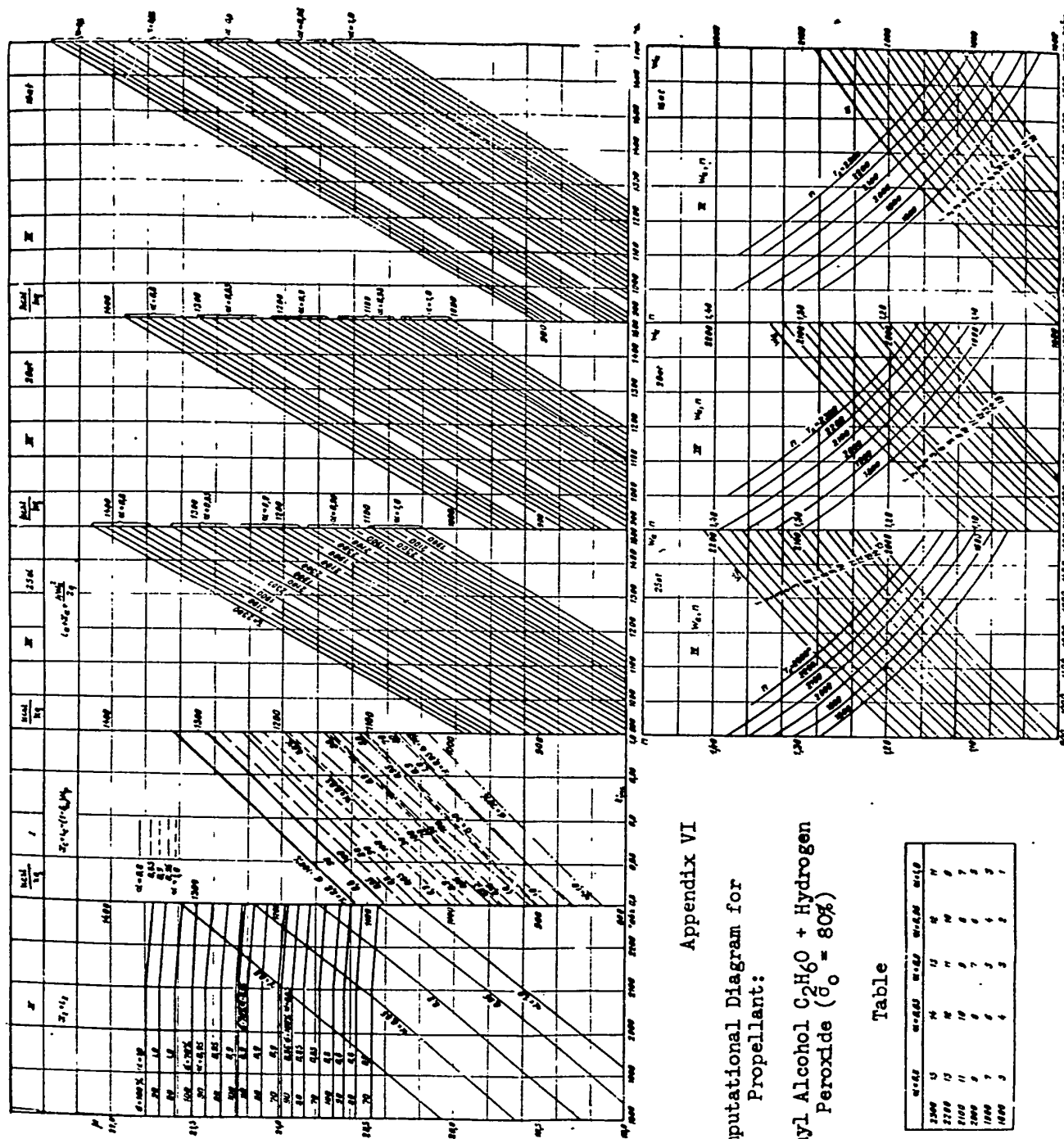
## Appendix IV

## Heat of Solution

in kcal/kg  $H_2O$ 

$H_2O$ in 96 %	$HNO_3$ . . . . .	326
$H_2O$ in 98 %	$HNO_3$ . . . . .	374
$H_2O$ in $N_2O_4$ . . . . .		0
$H_2O$ in $C_2H_5O$ . . . . .		14
$H_2O$ in $H_2O_2$ . . . . .		11

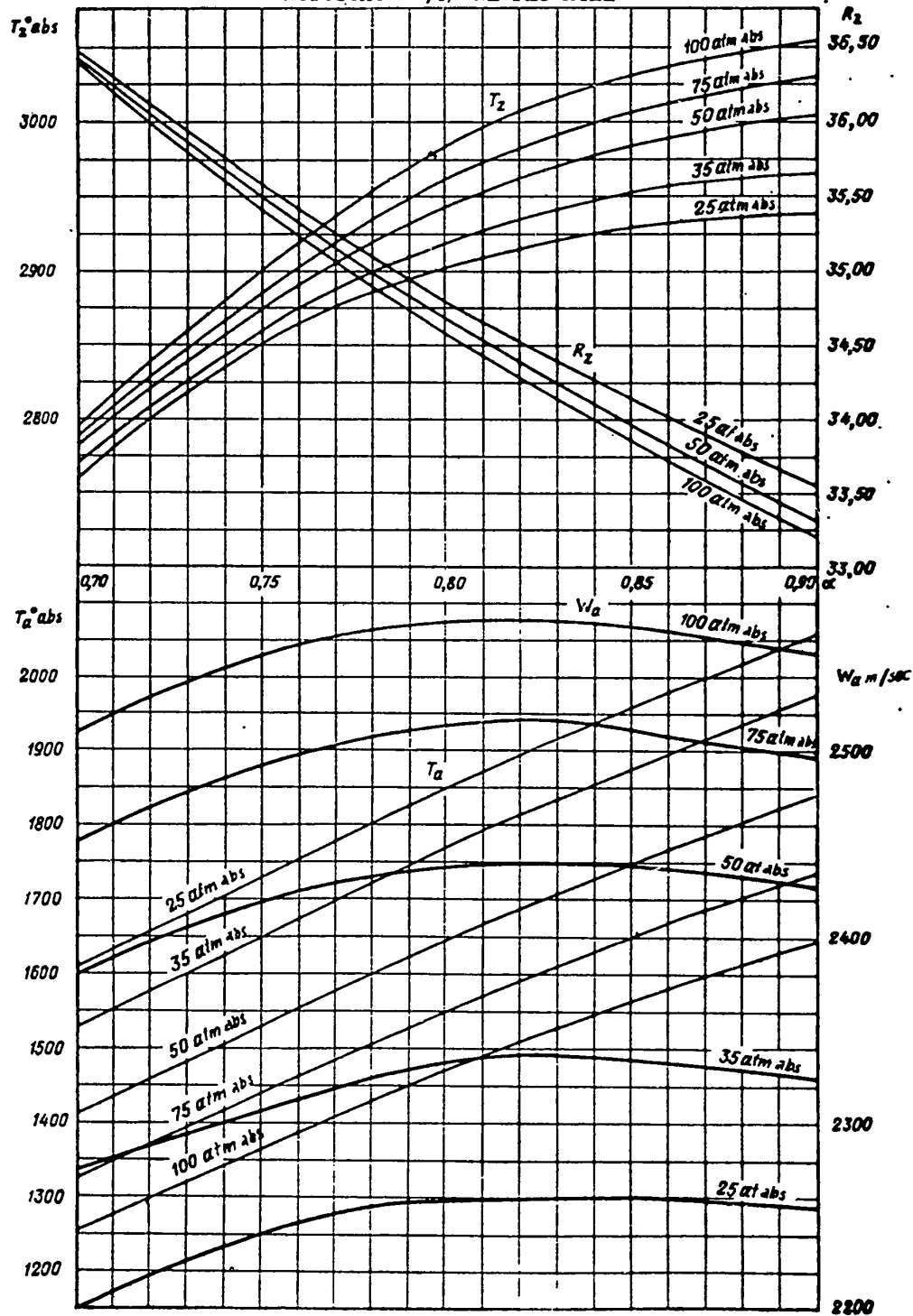




## Appendix VII

## Computational Diagram for Propellant:

Kerosene + 96% Nitric Acid





BIBLIOGRAPHY

1. Bolgarskiy, A.V. and Shchukin, V.K. - Working Processes in Liquid-Fuel Jet Engines.  
Oborongiz (1953), p.121

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